



CONSTRUCTION OF (a, b, c) TILINGS OF THE EUCLIDEAN PLANE, HYPERBOLIC PLANE AND THE SPHERE

MARK D. TOMENES AND MA. LOUISE ANTONETTE N. DE LAS PEÑAS

ABSTRACT. An (a, b, c) tiling forms under its symmetry group a orbits of vertices; b orbits of edges; and c orbits of tiles. This paper discusses a method to arrive at an (a, b, c) tiling of the Euclidean plane ($\mathbb{E}^2$), hyperbolic plane ($\mathbb{H}^2$) or 2-dimensional sphere ($\mathbb{S}^2$). An application of the method facilitates the complete enumeration of the $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ as well as a listing of $(a, 3, c)$ tilings of $\mathbb{E}^2$.

1. INTRODUCTION

One area of study in discrete geometry is the classification of tilings based on the vertex, edge or tile transitivity properties. In the literature, tilings are usually characterized and enumerated, focusing solely on isogonality (vertex) [13, 20, 23, 27], isotoxality (edge) [17, 19, 21, 24] or isohedrality (tile) properties [5, 11, 15, 16, 21, 22, 29, 30, 31].

Tilings that assume simultaneously, vertex, edge and tile transitivity conditions have not been studied extensively. In [1], Chavey introduced the notion of an (a, b, c) tiling, a tiling that forms a orbits of vertices, b orbits of edges and c orbits of tiles under its symmetry group. In the same study it was determined that not all triples $a - b - c$ can be realized as (a, b, c) tilings of the Euclidean plane ($\mathbb{E}^2$); and conditions relating a , b and c were specified. In [12], Dress and Scharlau derived 37 types of $(2, 2, 2)$ tilings of $\mathbb{E}^2$. Apart from this, there is still no complete enumeration of (a, b, c) tilings for $b > 1$ as of this writing. Also, there is hardly any literature on (a, b, c) tilings of the hyperbolic plane ($\mathbb{H}^2$).

In recent years, (a, b, c) tilings have appeared prominently in crystallography in accordance with design synthesis and reticular chemistry [7, 8, 9, 10].

Received by the editors August 21, 2022, and in revised form November 15, 2023.

2010 *Mathematics Subject Classification.* 52C20.

Key words and phrases. (a, b, c) tilings, tilings, transitivity, vertex-edge-tile transitive.

M. Tomenes would like to acknowledge the Philippine Department of Science and Technology (DOST) through the Accelerated Science and Technology Human Resource Development Program (ASTHRDP) and the University Research Council, Ateneo de Manila University for the financial support during his Ph.D. and postdoctoral studies, respectively.

This work is licensed under a Creative Commons “Attribution-NoDerivatives 4.0 International” license.



In [6], –Friedrichs and O’Keeffe noted that 2-periodic $(a, a - 1, c)$ tilings are rare. The authors presented seven $(3, 2, c)$ tilings of $\mathbb{E}^2$ and 12 $(3, 2, c)$ tilings of the 2-dimensional sphere ($\mathbb{S}^2$).

Considering these facts, the objective of this paper is to contribute to existing studies on (a, b, c) tilings. An approach to construct (a, b, c) tilings of $\mathbb{E}^2$, $\mathbb{H}^2$ or $\mathbb{S}^2$ is presented. The method facilitates arriving at tilings with a -isogonality, b -isotoxality and c -isohedrality properties, simultaneously. In this paper we give the complete enumeration of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$; and a listing of $(a, 3, c)$ tilings of $\mathbb{E}^2$. Moreover, new examples of (a, b, c) tilings of $\mathbb{H}^2$ are made possible, such as those arising from regular tilings $[p^n]$ and semi-regular tilings.

In the literature, most studies dealing with the construction and classification of tilings are approached combinatorially using adjacency symbols [16, 19, 20, 21], Delaney-Dress symbols [22] or generalized Schläfli symbols [12]. In this work, the approach employed is group theoretic, involving subgroup structures of symmetry groups of tilings, and orbit-stabilizer conditions.

This paper is organized as follows. Section 2 gives the basic definitions used in the paper. Section 3 presents a method of constructing (a, b, c) tilings. Section 4 and Section 5 outline, respectively, the enumeration of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$; and a listing of $(a, 3, c)$ tilings of $\mathbb{E}^2$. Section 6 discusses (a, b, c) tilings of $\mathbb{E}^2$ realized by convex tiles only. Section 7 highlights particular (a, b, c) tilings of $\mathbb{H}^2$. Finally, Section 8 gives the conclusion and future outlook of the study.

2. PRELIMINARIES

A *tiling* $\mathcal{T}$ of $\mathbb{X}$ ($\mathbb{X}$ is the Euclidean plane ($\mathbb{E}^2$), hyperbolic plane ($\mathbb{H}^2$) or 2-dimensional sphere ($\mathbb{S}^2$)) is a countable collection of closed topological disks called *tiles* $\mathcal{T} = \{T_i : i \in \mathbb{N}\}$ that is a covering ($\cup_i T_i = \mathbb{X}$) as well as a packing ($\text{Int}(T_i) \cap \text{Int}(T_j) = \emptyset$ if $i \neq j$, $\text{Int}(T)$ denotes the interior of tile T). The intersection of any two distinct tiles can be a set of isolated points and arcs. The points are called *vertices* of $\mathcal{T}$ and the arcs are called *edges* of $\mathcal{T}$. A vertex with p edges incident to it is said to have *valence* p . The tiling $\mathcal{T}$ is *edge-to-edge* if the intersection of any of its two tiles are either the common edge or vertex of the tiles, or empty.

The *symmetry group* G of $\mathcal{T}$ is the group of isometries of $\mathbb{X}$ that leave $\mathcal{T}$ invariant. The elements of G are called *symmetries* of $\mathcal{T}$.

Consider a vertex x , edge E or tile T of $\mathcal{T}$ and H a subgroup of G ($H \leq G$). The *stabilizer* of x (respectively E , T) in H denoted by $\text{Stab}_H(x)$ (respectively $\text{Stab}_H(E)$, $\text{Stab}_H(T)$) is defined as the group consisting of elements of H that fix x , that is, $\text{Stab}_H(x) = \{h \in H : hx = x\}$ (respectively $\text{Stab}_H(E) = \{h \in H : hE = E\}$, $\text{Stab}_H(T) = \{h \in H : hT = T\}$). For $H \leq G$, the set $Hx = \{hx : h \in H\}$ (respectively $HE = \{hE : h \in H\}$, $HT = \{hT : h \in H\}$) is defined as the *orbit* of x (respectively E , T) under

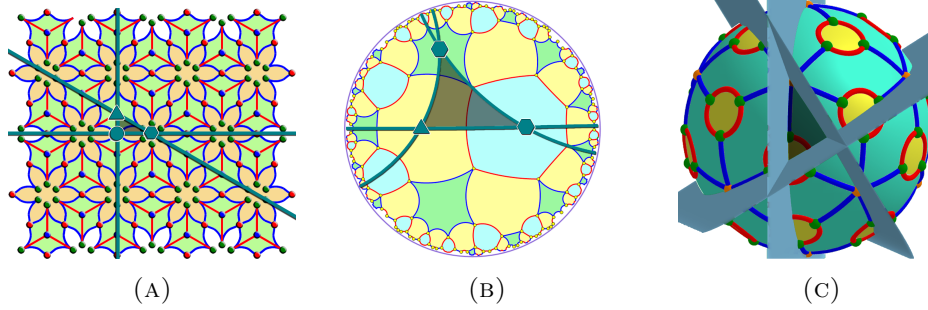


FIGURE 1. (A) $(3, 2, 2)$ tiling of $\mathbb{E}^2$; (B) $(1, 2, 3)$ tiling of $\mathbb{H}^2$; and (C) $(2, 2, 2)$ tiling of $\mathbb{S}^2$. The axes/planes of reflections that generate the symmetry group of a tiling are shown.

the action of H . A group G is said to *act transitively* on a set of vertices, edges or tiles of $\mathcal{T}$ if there is only one orbit of vertices, edges or tiles of $\mathcal{T}$, respectively, under G .

If the symmetry group G of a tiling $\mathcal{T}$ forms k orbits of vertices, edges or tiles of $\mathcal{T}$, then the tiling is called, respectively, *k-isogonal*, *k-isotoxal* or *k-isohedral*. If $k = 1$, then $\mathcal{T}$ is called an *isogonal*, *isotoxal* or *isohedral tiling*, respectively.

A tiling $\mathcal{T}$ of $\mathbb{X}$ is called an (a, b, c) tiling if $\mathcal{T}$ forms a , b and c orbits of vertices, edges and tiles, respectively, under the action of its symmetry group G . $\mathcal{T}$ is also called an *a-isogonal*, *b-isotoxal* and *c-isohedral tiling*. Not all triples $a - b - c$ can be realized as an (a, b, c) tiling. In [1], Chavey states that in an (a, b, c) tiling, $a \leq b + 1$ and $c \leq b + 1$.

The tilings under consideration in this study are edge-to-edge tilings with tiles that are closed and bounded topological disks. We assume the tilings are such that the intersection of two tiles is a connected set. We exclude tilings with digons (tiles with two edges) or tilings with vertices of valence two.

In this paper, the symmetry group of a tiling is described using Conway's orbifold notation, which is based on the type of symmetries occurring in the group [3, 2]. The symbol $*$ denotes a reflection, $\times$ a glide reflection or roto-reflection, $\circ$ a translation and a positive integer u indicates a u -fold rotation. If u comes after $*$, the symmetry there is dihedral of order $2u$ (e.g. center of rotation lies on a reflection axis). For tilings of $\mathbb{E}^2$ or $\mathbb{H}^2$, a center of rotation of order n , $n > 2$ will be labeled by an n -gon. A twofold rotation will be labeled by an oval.

Illustration 2.1. A $(3, 2, 2)$ tiling of $\mathbb{E}^2$ is shown in Figure 1A. There are three orbits of vertices (blue, red and green), two orbits of edges (red and blue) and two orbits of tiles (green and orange) under its symmetry group $G_1^* = \langle P_1, Q_1, R_1 | P_1^2 = Q_1^2 = R_1^2 = (P_1 R_1)^2 = (P_1 Q_1)^3 = (Q_1 R_1)^6 \rangle \cong *632$. The tiling of $\mathbb{H}^2$ shown in Figure 1B forms one orbit of vertices (yellow), two

orbits of edges (blue and red) and three orbits of tiles (yellow, blue and green) under its symmetry group $G_2^* = \langle P_2, Q_2, R_2 | P_2^2 = Q_2^2 = R_2^2 = (P_2 R_2)^3 = (P_2 Q_2)^6 = (Q_2 R_2)^6 \rangle \cong *663$. Hence, it is a $(1, 2, 3)$ tiling. Finally, the tiling of $\mathbb{S}^2$ in Figure 1C is a $(2, 2, 2)$ tiling since it forms two orbits of vertices (orange and green), two orbits of edges (blue and red) and two orbits of tiles (blue and yellow) under its symmetry group $G_3^* = \langle P_3, Q_3, R_3 | P_3^2 = Q_3^2 = R_3^2 = (P_3 R_3)^3 = (P_3 Q_3)^5 = (Q_3 R_3)^2 \rangle \cong *532$.

3. METHOD OF CONSTRUCTING (a, b, c) TILINGS

Theorem 3.1. *Let $\mathcal{T}$ be an edge-to-edge tiling of $\mathbb{X}$ (Euclidean plane ($\mathbb{E}^2$), hyperbolic plane ($\mathbb{H}^2$) or 2-dimensional sphere ($\mathbb{S}^2$)) with symmetry group G . Let $H \leq G$. If H forms $\bar{r}$ orbits of tiles, $\bar{s}$ orbits of edges and $\bar{t}$ orbits of vertices in $\mathcal{T}$ then there exists an (a, b, c) tiling $\mathcal{T}^*$ of $\mathbb{X}$ satisfying $a \leq \bar{r}$, $b \leq \bar{s}$ and $c \leq \bar{t}$ with symmetry group G^* such that $H \leq G^*$.*

Proof. Consider $\mathcal{T}$, an edge-to-edge tiling of $\mathbb{X}$. Let $H \leq G$, G the symmetry group of $\mathcal{T}$. Suppose H forms $\bar{r}$ orbits of tiles, $\bar{s}$ orbits of edges and $\bar{t}$ orbits of vertices in $\mathcal{T}$. For each orbit HT_i , ($i = 1, \dots, \bar{r}$), consider a tile $T_i \in HT_i$, that is, T_i is an orbit representative of HT_i . Then, take a point $x_i \in \text{Int}(T_i)$ such that $hx_i = x_i \forall h \in \text{Stab}_H(T_i)$ and form $\mathcal{V} = Hx_1 \cup Hx_2 \cup \dots \cup Hx_{\bar{r}}$. This will result in every tile of $\mathcal{T}$ containing exactly one point from $\mathcal{V}$.

Take an edge $E_j \in HE_j$ ($j = 1, \dots, \bar{s}$). Consider $x_{j,1} \in \text{Int}(T_{j,1}) \cap \mathcal{V}$, $x_{j,2} \in \text{Int}(T_{j,2}) \cap \mathcal{V}$, where $T_{j,1}, T_{j,2}$ are the tiles incident to E_j . Connect $x_{j,1}$ and $x_{j,2}$ by an edge E_j^* such that $hE_j^* = E_j^* \forall h \in \text{Stab}_H(E_j)$. Form $HE_1^* \cup HE_2^* \cup \dots \cup HE_{\bar{s}}^*$. This is the skeleton of an edge-to-edge tiling $\mathcal{T}^*$ where pairs of points from $\mathcal{V}$ lying on tiles which are adjacent to an edge of $\mathcal{T}$ are joined to form edges of $\mathcal{T}^*$.

The vertices of $\mathcal{T}^*$ are elements of $\mathcal{V}$ and its tiles are d -gons, $d > 2$. A vertex V of $\mathcal{T}$ with valence d will give rise to a d -gonal tile T^* in $\mathcal{T}^*$, $V \in \text{Int}(T^*)$.

Since $H\mathcal{T}^* = \mathcal{T}^*$, it follows that $H \leq G^*$, where G^* is the symmetry group of $\mathcal{T}^*$.

If $G^* = H$, then $\mathcal{T}^*$ is a $(\bar{r}, \bar{s}, \bar{t})$ tiling since $\mathcal{V} = Hx_1 \cup Hx_2 \cup \dots \cup Hx_{\bar{r}}$ and $\mathcal{T}^* = HE_1^* \cup HE_2^* \cup \dots \cup HE_{\bar{s}}^*$. Moreover, since every tile of $\mathcal{T}^*$ contains a vertex of $\mathcal{T}$, and H forms $\bar{t}$ orbits of vertices in $\mathcal{T}$, then it follows that H forms $\bar{t}$ orbits of tiles in $\mathcal{T}^*$.

Suppose $G^* > H$. If $g_1 x_i^* = x_j^*$ for some $g_1 \in G^* \setminus H$, $x_i^* \in Hx_i$, $x_j^* \in Hx_j$, x_i^*, x_j^* vertices of $\mathcal{T}^*$, $i, j \in \{1, 2, \dots, \bar{r}\}$, $i \neq j$, $\bar{r} > 1$, then there are a orbits of vertices under G^* where $a < \bar{r}$. If $g_2 E_k' = E_l'$ for some $g_2 \in G^* \setminus H$, $E_k' \in HE_k^*$, $E_l' \in HE_l^*$, E_k', E_l' edges of $\mathcal{T}^*$, $k, l \in \{1, 2, \dots, \bar{s}\}$, $k \neq l$, $\bar{s} > 1$, then there are b orbits of vertices under G^* where $b < \bar{s}$. Finally, if $g_3 T_m^* = T_n^*$ for some $g_3 \in G^* \setminus H$, $T_m^* \in HT_m^*$, $T_n^* \in HT_n^*$, T_m^*, T_n^* tiles of $\mathcal{T}^*$, $m, n \in \{1, 2, \dots, \bar{t}\}$, $m \neq n$, $\bar{t} > 1$, then there are c orbits of tiles under G^* where $c < \bar{t}$. If $\bar{r} = 1$, $\bar{s} = 1$ or $\bar{t} = 1$, then we have $a = \bar{r}$, $b = \bar{s}$ or $c = \bar{t}$ respectively. Thus, $\mathcal{T}^*$ is an (a, b, c) tiling where $a \leq \bar{r}$, $b \leq \bar{s}$, $c \leq \bar{t}$. $\square$

We assume in Theorem 3.1 that the symmetry group G of the tiling $\mathcal{T}$ is a plane crystallographic group for a tiling of $\mathbb{E}^2$ or $\mathbb{H}^2$, or a finite spherical group for a tiling of $\mathbb{S}^2$. We assume that the vertices, edges and tiles of $\mathcal{T}$ have stabilizers C_d or D_d (cyclic group of order d or dihedral group of order $2d$, respectively) in G . Consequently, as a result of the construction, the symmetry group G^* of the resulting tiling $\mathcal{T}^*$ is either a plane crystallographic group or a finite spherical group. The vertices, edges and tiles of $\mathcal{T}^*$ have stabilizers C_d or D_d in G^* .

An edge E_j^* ($j = 1, \dots, \bar{s}$) used in the construction of $\mathcal{T}^*$ in Theorem 3.1 may be one of five types, shown in Figure 2.

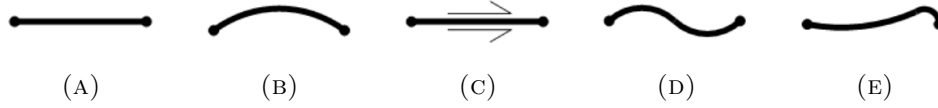


FIGURE 2. Edge E_j^* with (A) $\text{Stab}_H(E_j^*) \cong D_2$; (B)-(C) $\text{Stab}_H(E_j^*) \cong D_1$; (D) $\text{Stab}_H(E_j^*) \cong C_2$; and (E) $\text{Stab}_H(E_j^*) \cong C_1$.

Illustration 3.2. We show the application of Theorem 3.1 via a construction of a $(4, 3, 2)$ tiling of $\mathbb{E}^2$. We consider a starting tiling $\mathcal{T}$ of $\mathbb{E}^2$ consisting of regular 4-gons with symmetry group $G = \langle P, Q, R \rangle \cong *442$ (Figure 3A). Take the subgroup $H = \langle Q, R, PQPRQRPQP \rangle \cong *442$ of G where $[G : H] = 8$. Under H , $\mathcal{T}$ forms four orbits of tiles, three orbits of edges and two orbits of vertices ($\bar{r} = 4$, $\bar{s} = 3$, $\bar{t} = 2$). Consider tiles $T_1 \in HT_1$, $T_2 \in HT_2$, $T_3 \in HT_3$, $T_4 \in HT_4$ (Figure 3B). Choose $x_1 \in \text{Int}(T_1)$, $x_2 \in \text{Int}(T_2)$, $x_3 \in \text{Int}(T_3)$, $x_4 \in \text{Int}(T_4)$, satisfying $h_1x_1 = x_1 \forall h_1 \in \text{Stab}_H(T_1) = \langle Q, R \rangle \cong D_4$; $h_2x_2 = x_2 \forall h_2 \in \text{Stab}_H(T_2) = \langle R \rangle \cong D_1$; $h_3x_3 = x_3 \forall h_3 \in \text{Stab}_H(T_3) = \langle Q, PQPRQRPQP \rangle \cong D_2$; $h_4x_4 = x_4 \forall h_4 \in \text{Stab}_H(T_4) = \langle R, PQPRQRPQP \rangle \cong D_4$ (Figure 3C). Then we form $\mathcal{V} = Hx_1 \cup Hx_2 \cup Hx_3 \cup Hx_4$ (Figure 3D), consisting of the vertices of the tiling we are constructing.

Now, consider $E_1 \in HE_1$, $E_2 \in HE_2$, $E_3 \in HE_3$ (Figure 3E). Consider the vertices $x_{1,1} \in \text{Int}(T_{1,1}) \cap \mathcal{V}$ and $x_{1,2} \in \text{Int}(T_{1,2}) \cap \mathcal{V}$ where $T_{1,1}$ and $T_{1,2}$ are tiles incident to E_1 . Connect $x_{1,1}$ and $x_{1,2}$ by an edge E_1^* satisfying $h_1E_1^* = E_1^* \forall h_1 \in \text{Stab}_H(E_1) = \langle R \rangle \cong D_1$ (Figure 3F). In a similar manner, edges E_2^* and E_3^* are constructed satisfying $h_2E_2^* = E_2^* \forall h_2 \in \text{Stab}_H(E_2) \cong C_1$; $h_3E_3^* = E_3^* \forall h_3 \in \text{Stab}_H(E_3) = \langle R \rangle \cong D_1$ (Figure 3G). Then we form $\mathcal{T}^* = HE_1^* \cup HE_2^* \cup HE_3^*$ (Figure 3H). $\mathcal{T}^*$ is an edge-to-edge $(4, 3, 2)$ tiling with symmetry group $G^* = H = \langle Q, R, PQPRQRPQP \rangle \cong *442$.

The next two results will be helpful in describing (a, b, c) tilings obtained from the previous theorem.

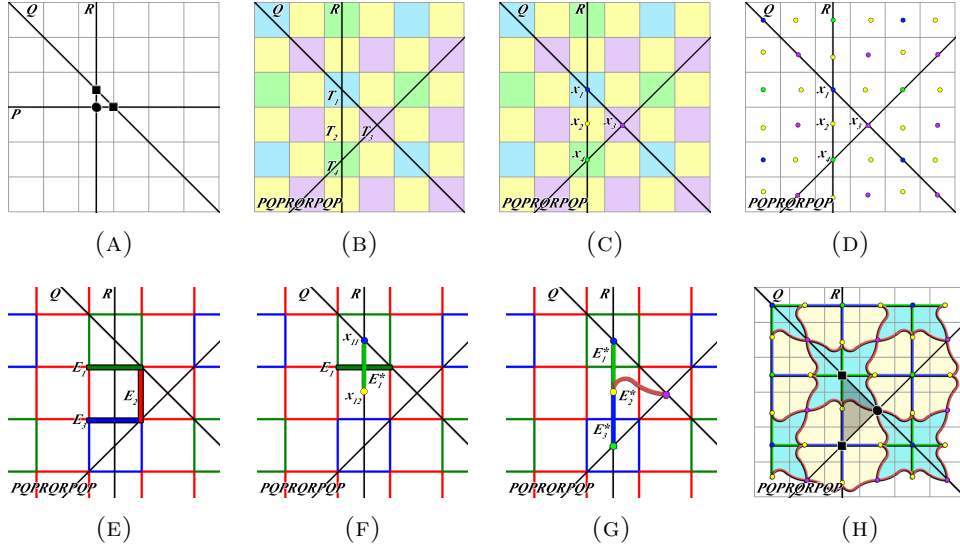


FIGURE 3. Construction of a $(4, 3, 2)$ tiling $\mathcal{T}^*$ from the tiling $\mathcal{T}$ by 4-gons: (A) $\mathcal{T}$ with axes of reflections P, Q, R ; (B) tile orbit representatives under $H = \langle Q, R, PQPRQRPQP \rangle \cong *442$; (C) vertices $x_1 \in \text{Int}(T_1)$, $x_2 \in \text{Int}(T_2)$, $x_3 \in \text{Int}(T_3)$, $x_4 \in \text{Int}(T_4)$; (D) $\mathcal{V} = Hx_1 \cup Hx_2 \cup Hx_3 \cup Hx_4$; (E) edge orbit representatives E_1, E_2, E_3 under H ; (F) edge E_1^* where $\text{Stab}_H(E_1^*) = \langle R \rangle \cong D_1$; (G) edges E_2^* and E_3^* where $\text{Stab}_H(E_2^*) \cong C_1$ and $\text{Stab}_H(E_3^*) = \langle R \rangle \cong D_1$; and (H) $(4, 3, 2)$ tiling $\mathcal{T}^* = HE_1^* \cup HE_2^* \cup HE_3^*$.

Theorem 3.3. Suppose G acts transitively on the set $\mathcal{O}$ of tiles, edges or vertices of a tiling of $\mathbb{X}$. Let $H \leq G$. If H forms n orbits of tiles, edges or vertices in $\mathcal{O}$ and $\text{Stab}_G(x)$ is finite, then

$$[G : H] = \sum_{i=1}^n \frac{|\text{Stab}_G(x)|}{|\text{Stab}_H(x_i)|}$$

where x_i , $i = 1, \dots, n$, is a representative of an H orbit of vertex, edge or tile and $x \in \mathcal{O}$.

Proof. Since H forms n orbits of tiles, edges or vertices in $\mathcal{O}$ then $\mathcal{O} = Hx_1 \cup Hx_2 \cup \dots \cup Hx_n$, where $x_i \in \mathcal{O}$, $i = 1, \dots, n$.

Let $x \in \mathcal{O}$. Since G acts transitively on $\mathcal{O}$, for all $x_i^j \in Hx_i$, $j = 1, 2, \dots$, there exists $g_i^j \in G$ such that $g_i^j x = x_i^j$. We now show that the set $\{g \in G \mid gx = x_i^j\}$ is the coset $g_i^j S$ where $S = \text{Stab}_G(x)$. Consider $g^* \in \{g \in G \mid gx = x_i^j\}$. We can write g^* as $g^* = (g_i^j)(g_i^j)^{-1}g^*$ where $(g_i^j)^{-1}g^* \in S$ since $(g_i^j)^{-1}g^*x = (g_i^j)^{-1}x_i^j = x$ so that $g^* \in g_i^j S$. Conversely, if $g^* \in g_i^j S$ then $g^* = g_i^j g$, $g \in S$. Since $g^*x = g_i^j gx = g_i^j x = x_i^j$ then $g^* \in \{g \in G \mid gx = x_i^j\}$.

Construct the set $X_i = \cup_{j=1}^{\infty} g_i^j S$. We show $\cup_{i=1}^n X_i = G$. If $g^* \in \cup_{i=1}^n X_i = \cup_{i=1}^n \cup_{j=1}^{\infty} g_i^j S$, then $g^* \in G$. Conversely, if $g^* \in G$, then $g^* x \in Hx_i$ for some $i \in \{1, 2, \dots, n\}$. Thus, $g^* x = x_i^j$ for some $j \in \{1, 2, \dots\}$ and hence $g^* \in \{g \in G | gx = x_i^j\} = g_i^j S$. Consequently, $g^* \in X_i$ and therefore, $\cup_{i=1}^n X_i = G$.

The number of elements of H in S is $|S_1|$ where $S_1 = \text{Stab}_H(x_1)$ and since H acts transitively on Hx_1 , H must also contain $|S_1|$ elements from each coset $g_1^j S$ in X_1 . This results to $|S|/|S_1|$ cosets of H in G . In general, for each X_i , we obtain $|S|/|S_i|$ cosets of H in G . Since the union of all cosets in $X_1, X_2, \dots, X_n$ is G , we obtain

$$\sum_{i=1}^n \frac{|S|}{|S_i|} = \sum_{i=1}^n \frac{|\text{Stab}_G(x)|}{|\text{Stab}_H(x_i)|}$$

cosets of H in G . □

Corollary 3.4. *Under the conditions that Theorem 3.3 holds, if $x \in \mathcal{O}$, then $n \leq [G : H] \leq n|\text{Stab}_G(x)|$.*

Proof. We have $1 \leq |\text{Stab}_H(x_i)| \leq |\text{Stab}_G(x)|$ for all $i = 1, \dots, n$. If $|\text{Stab}_H(x_i)| = 1$ for all i , then $[G : H]$ attains its maximum value which is $n|\text{Stab}_G(x)|$. On the other hand, if $|\text{Stab}_H(x_i)| = |\text{Stab}_G(x)|$ for all i then $[G : H]$ attain its minimum value which is n . Thus, $n \leq [G : H] \leq n|\text{Stab}_G(x)|$. □

In Theorem 3.1, the following result is true.

Theorem 3.5. *Let $\mathcal{T}$ be an $(\bar{a}, \bar{b}, \bar{c})$ tiling. If H forms a orbits of vertices, b orbits of edges and c orbits of tiles in $\mathcal{T}^*$ then*

$$(3.1) \quad a \leq \bar{c}[G : H] \leq (a - \bar{c} + 1) \sum_{i=1}^{\bar{c}} |\text{Stab}_G(\bar{T}_i)|;$$

$$(3.2) \quad b \leq \bar{b}[G : H] \leq (b - \bar{b} + 1) \sum_{j=1}^{\bar{b}} |\text{Stab}_G(\bar{E}_j)|; \text{ and}$$

$$(3.3) \quad c \leq \bar{a}[G : H] \leq (c - \bar{a} + 1) \sum_{k=1}^{\bar{a}} |\text{Stab}_G(V_k)|$$

where $\bar{T}_i$, $i = 1, \dots, \bar{c}$ are tile representatives in each orbit of tiles, $\bar{E}_j$, $j = 1, \dots, \bar{b}$ are edge representatives in each orbit of edges, V_k , $k = 1, \dots, \bar{a}$ are vertex representatives in each orbit of vertices under G .

Proof. We prove only (3.1). The inequalities (3.2) and (3.3) can be shown using a similar argument. Since $\mathcal{T}$ has $\bar{c}$ orbits of tiles under G , we can write $\mathcal{T} = G\bar{T}_1 \cup G\bar{T}_2 \cup \dots \cup G\bar{T}_{\bar{c}}$ where $\bar{T}_i$, $i = 1, \dots, \bar{c}$ are tile orbit representatives.

Suppose the tiles in $G\bar{T}_i$ form n_i orbits under H , that is, $G\bar{T}_i = H\bar{T}_{i,1} \cup H\bar{T}_{i,2} \cup \dots \cup H\bar{T}_{i,n_i}$ where $\bar{T}_{i,j}$, $j = 1, 2, \dots, n_i$, are tile representatives in

each orbit in $G\bar{T}_i$ under H . Thus, $\mathcal{T}$ has $\sum_{i=1}^{\bar{c}} n_i$ orbits of tiles under H . Since $\mathcal{T}^*$ has a orbits of vertices under H , it follows that $a = \sum_{i=1}^{\bar{c}} n_i$.

Now, since G acts transitively on $G\bar{T}_i$ and $G\bar{T}_i$ forms n_i orbits under H , then by Corollary 3.4, we have $n_i \leq [G : H] \leq n_i |\text{Stab}_G(\bar{T}_i)|$. Thus,

$$(3.4) \quad \sum_{i=1}^{\bar{c}} n_i \leq \bar{c}[G : H] \leq \sum_{i=1}^{\bar{c}} n_i |\text{Stab}_G(\bar{T}_i)|.$$

Since $n_i - 1 \leq \sum_{i=1}^{\bar{c}} (n_i - 1)$ for all $i = 1, 2, \dots, \bar{c}$, and $a = \sum_{i=1}^{\bar{c}} n_i$, it follows that $n_i \leq a - \bar{c} + 1$. Hence, (3.4) becomes

$$a \leq \bar{c}[G : H] \leq (a - \bar{c} + 1) \sum_{i=1}^{\bar{c}} |\text{Stab}_G(\bar{T}_i)|.$$

□

Remark 3.6. From (3.1)-(3.3), we obtain, respectively, $\bar{c} \leq a$, $\bar{b} \leq b$, $\bar{a} \leq c$.

4. $(a, 2, c)$ TILINGS OF $\mathbb{E}^2$ AND $\mathbb{S}^2$

In this part of the paper, we apply the results given in the previous section to arrive at $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$.

In the construction of an $(a, 2, c)$ tiling we have $\bar{b} \leq 2$ by Remark 3.6 so the starting tiling $\mathcal{T}$ to consider has one or two orbits of edges under its symmetry group, that is, $\mathcal{T}$ is isotoxal or 2-isotoxal. It is ideal to use a homeotoxal or a 2-homeotoxal tiling provided in [17] as the starting tiling $\mathcal{T}$. Recall that a tiling of $\mathbb{E}^2$ (or $\mathbb{S}^2$) is said to be *homeotoxal* or *2-homeotoxal* if the group containing the homeomorphisms of $\mathbb{E}^2$ (or $\mathbb{S}^2$) that map the tiling onto itself, form one orbit or two orbits of edges, respectively. Note that if the edges of a 2-isotoxal tiling form two transitivity classes under its symmetry group, then the edges form one or two transitivity classes under the group of homeomorphisms sending the tiling to itself. In which case, a 2-isotoxal tiling is necessarily either homeotoxal or 2-homeotoxal [17]. Similarly, an isotoxal tiling is necessarily a homeotoxal tiling.

The entire process in obtaining the complete list of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ is outlined below.

- (1) We take each of the homeotoxal and 2-homeotoxal tilings provided in [17] as a starting tiling $\mathcal{T}$.
- (2) Given a starting tiling $\mathcal{T}$ with automorphism group G , we consider every subgroup H of G , distinct up to conjugacy, satisfying $2 \leq \bar{b}[G : H] \leq (3 - \bar{b}) \sum_{j=1}^{\bar{b}} |\text{Stab}_G(\bar{E}_j)|$. We use the computer software GAP [14] in arriving at the subgroups of G .

If $\mathcal{T}$ is a homeotoxal tiling, then $\bar{b} = 1$ and we take every subgroup H of G satisfying $2 \leq [G : H] \leq 2|\text{Stab}_G(E_1)|$. On the other hand, if $\mathcal{T}$ is a 2-homeotoxal tiling, then $\bar{b} = 2$ and we take every subgroup H of G satisfying $1 \leq [G : H] \leq (|\text{Stab}_G(E_1)| + |\text{Stab}_G(E_2)|)/2$.

- (3) For every subgroup H in the list of subgroups obtained in Step 2, we check if it forms two orbits of edges in $\mathcal{T}$ ($\bar{s} = 2$) to be assured that we will get an $(a, 2, c)$ tiling when applying Theorem 3.1. If $\bar{s} = 2$, then we proceed with the method given in Theorem 3.1 paragraph 2 and construct $\mathcal{T}^* = HE_1^* \cup HE_2^*$. In the case of $\mathcal{T}$ a homeotoxal tiling, the edges E_1^* and E_2^* must be carefully selected such that $gE_1^* \neq E_2^*$, where $g \in G \setminus H$ so as not to obtain an $(a, 1, c)$ tiling.

From the process outlined above, of taking all the homeotoxal and 2-homeotoxal tilings, and considering all the qualified subgroups (distinct up to conjugacy) of the automorphism group of a homeotoxal or two homeotoxal tiling, we obtain the complete list of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ distinct up to homeomerism. The results are presented in Theorem 4.1 and Table 1 for $\mathbb{E}^2$, and in Theorem 4.3 and Tables 2 and 3 for $\mathbb{S}^2$. The tilings with tiles that have disconnected intersections are not included.

We recall that two tilings of $\mathbb{E}^2$ (or $\mathbb{S}^2$) are said to be *homeomeric* provided there exists a homeomorphism of $\mathbb{E}^2$ (or $\mathbb{S}^2$) mapping one tiling onto the other, which maps each symmetry of either tiling onto a symmetry of the other tiling. Each homeomeric type of $(a, 2, c)$ tiling appearing in Table 1 and Table 2 is described by its

- a) symmetry group; and
- b) *edge transitivity symbol* given by

$$\mathcal{S} := \langle m_1^{S_{11}}.n_1^{S_{12}}; p_1^{S_{13}}.q_1^{S_{14}} \rangle_{S_1} \langle m_2^{S_{21}}.n_2^{S_{22}}; p_2^{S_{23}}.q_2^{S_{24}} \rangle_{S_2}$$

where $\langle m_j^{S_{j1}}.n_j^{S_{j2}}; p_j^{S_{j3}}.q_j^{S_{j4}} \rangle_{S_j}$ describes an edge orbit representative E_j^* , $j = 1, 2$. E_j^* is adjacent to an m_j -gon and an n_j -gon, and the endpoints (vertices) have valences p_j and q_j . A superscript S_{jl} , $l = 1, \dots, 4$ of m_j, n_j (respectively p_j, q_j) is either D_d (dihedral group of order $2d$) or C_d (cyclic group of order d) denoting the group type of the stabilizer of the corresponding tile (respectively, endpoint) in G^* . The superscript S_j indicates the group type of $\text{Stab}_{G^*}(E_j^*)$ which is either D_d or C_d .

Each subgroup (distinct up to conjugacy) from the list in step 2 that forms two orbits of edges in $\mathcal{T}$ yields an $(a, 2, c)$ tiling $\mathcal{T}^*$ with symmetry group $G^* = H$. This provides the information for a). The values for m, n, p, q in the edge transitivity symbol point to the homeotoxal tiling or 2-homeotoxal tiling used as the starting tiling $\mathcal{T}$. Both a) and b) are necessary to describe a homeomeric type of $(a, 2, c)$ tiling completely.

Two $(a, 2, c)$ tilings may have the same edge transitivity symbol, but are not homomeric because the tilings have distinct symmetry group types. This is the case, for instance, for the $(1, 2, 1)$ tilings no. 6 and no. 7 in Table 1 which share the same edge transitivity symbol and have symmetry groups of different Conway types. The Conway notation is the first criteria to differentiate between symmetry group types, but for the situation where

two tilings have symmetry groups with the same Conway notation, the positions of the fixed points and fixed lines of the distinct symmetries in the corresponding fundamental regions may serve as a guide to distinguish between the symmetry group types of the two tilings. To illustrate this point, the $(1, 2, 1)$ tilings no. 5 and no. 6 in Table 1 have the same edge transitivity symbol, and have the same Conway symmetry group type, namely, $\times \times$. For tiling no. 5, each axis of glide reflection in the fundamental region shown passes through edges belonging to the same edge orbit (Figure 4A). For tiling no. 6, only one of the axes of glide reflections in the fundamental region passes through edges belonging to the same edge orbit (Figure 4bB). In tiling no. 5, there are two adjacent edges belonging to the same tile that are sent to each other by a glide reflection, and in tiling no. 6, there are no two adjacent edges belonging to the same tile that are sent to each other by a glide reflection. Thus, these tilings are not homeomeric.

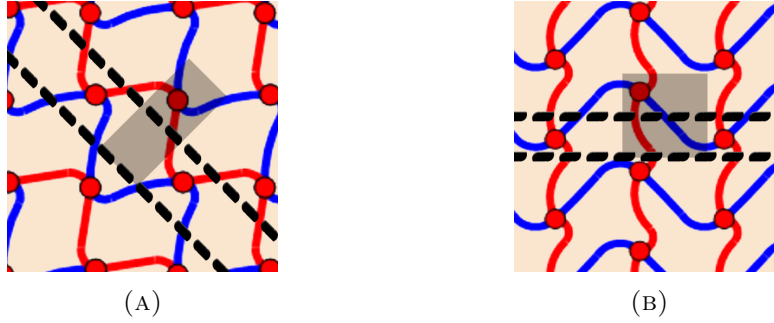


FIGURE 4. Non homeomeric $(1, 2, 1)$ tilings with symmetry group $\times \times$ and edge transitivity symbol $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1} \langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$.

We now present the result on $(a, 2, c)$ tilings of $\mathbb{E}^2$. The tilings are listed in Table 1 with their respective symmetry groups and edge transitivity symbols.

Theorem 4.1. *There are 104 $(a, 2, c)$ tilings of $\mathbb{E}^2$ distinct up to homeomorphism. Among these there are 15 $(1, 2, 1)$ tilings, 12 $(1, 2, 2)$ tilings, seven $(1, 2, 3)$ tilings, 12 $(2, 2, 1)$ tilings, 34 $(2, 2, 2)$ tilings, eight $(2, 2, 3)$ tilings, seven $(3, 2, 1)$ tilings, eight $(3, 2, 2)$ tilings and one $(3, 2, 3)$ tiling of $\mathbb{E}^2$.*

We give an example of the spherical case.

Illustration 4.2. Figure 5 shows the construction of $(1, 2, 2)$ tilings from the infinite sequence of tilings (2-homeotoxal tiling 2HTS53(n)) $\mathcal{T}_n := \langle 3^2; 4.n \rangle \langle 3^2; 4^2 \rangle$ for $n = 3, 5, 6$ (Figure 5A) with symmetry group $G_n = \langle P, Q, R \rangle \cong *n22$. As $\text{Stab}_{G_n}(E_1) \cong D_1$ and $\text{Stab}_{G_n}(E_2) \cong D_2$, where E_1, E_2 are edge orbit representatives under G_n , we choose subgroups H_n such that $1 \leq [G_n : H_n] \leq 3$. In particular, we take $H_n = \langle QP, RP \rangle \cong n22$, $[G_n : H_n] = 2$ (Figure 5B), where H_n forms two orbits of edges in $\mathcal{T}_n$. Figures

5C-5D show the construction of the $(1, 2, 2)$ tilings $\mathcal{T}_n^* = H_n E_1^* \cup H_n E_2^*$ with symmetry group $G_n^* = H_n = \langle QP, RP \rangle \cong n22$ and edge transitivity symbol $\mathcal{S}_n := \langle 4^{C_2}.n^{C_n}; 3^{C_1}.3^{C_1} \rangle^{C_1} \langle 4^{C_2}.4^{C_2}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ (Figure 5E).

The following theorem enumerates the $(a, 2, c)$ tilings of $\mathbb{S}^2$. The tilings are listed in Tables 2-3 with their respective symmetry groups and edge transitivity symbols.

Theorem 4.3.

- i) *There are 16 infinite sequences (each depending on a positive integer n) of $(a, 2, c)$ tilings of $\mathbb{S}^2$. Among these sequences, five are $(1, 2, 2)$ tilings, five are $(2, 2, 1)$ tilings, four are $(2, 2, 2)$ tilings, one is a $(2, 2, 3)$ tiling and one is a $(3, 2, 2)$ tiling of $\mathbb{S}^2$.*
- ii) *In addition to i), there are 82 $(a, 2, c)$ tilings of $\mathbb{S}^2$. Among these there are two $(1, 2, 1)$ tilings, 12 $(1, 2, 2)$ tilings, six $(1, 2, 3)$ tilings, 12 $(2, 2, 1)$ tilings, 24 $(2, 2, 2)$ tilings, ten $(2, 2, 3)$ tilings, six $(3, 2, 1)$ tilings and ten $(3, 2, 2)$ tilings of $\mathbb{S}^2$.*

5. $(a, 3, c)$ TILINGS OF $\mathbb{E}^2$

In this section, we present a list of $(a, 3, c)$ tilings. These have been arrived at from homeotoxal and 2-homeotoxal tilings from [17]. Moreover, we also considered possible starting tilings from the list of $\bar{a}$ -uniform tilings from [13]. These are $(\bar{a}, \bar{b}, \bar{c})$ tilings by regular polygons where $\bar{a} \leq 4$.

In arriving at the $(a, 3, c)$ tilings of $\mathbb{E}^2$, we follow a similar process used for obtaining the $(a, 2, c)$ tilings. Given a starting tiling (any of the aforementioned types) with an automorphism or symmetry group G , we consider every subgroup H of G , distinct up to conjugacy, that satisfies $3 \leq \bar{b}[G : H] \leq (3 - \bar{b} + 1) \sum_{j=1}^{\bar{b}} |\text{Stab}_G E_j|$, where E_j is an edge orbit representative; and such that H forms three orbits of edges in $\mathcal{T}$. In constructing the tiling $\mathcal{T}^*$, the edges E_1^*, E_2^*, E_3^* must be carefully selected such that $gE_i^* \neq E_j^*$, where $g \in G \setminus H$ and $i, j \in \{1, 2, 3\}$, $i \neq j$ to ensure deriving an $(a, 3, c)$ tiling.

The edge transitivity symbol for an $(a, 3, c)$ tiling is $\mathcal{S} := \langle m_1^{S_{11}}.n_1^{S_{12}}; p_1^{S_{13}}.q_1^{S_{14}} \rangle^{S_1} \langle m_2^{S_{21}}.n_2^{S_{22}}; p_2^{S_{23}}.q_2^{S_{24}} \rangle^{S_2} \langle m_3^{S_{31}}.n_3^{S_{32}}; p_3^{S_{33}}.q_3^{S_{34}} \rangle^{S_3}$. The description of the symbol follows that of an $(a, 2, c)$ tiling described in the previous section.

We now present our result on $(a, 3, c)$ tilings of $\mathbb{E}^2$ obtained from the method described above. Each homeomeric type of $(a, 3, c)$ tiling is described by its symmetry group and edge transitivity symbol. The tilings are listed in Table 4.

Theorem 5.1. *There exist 170 $(a, 3, c)$ tilings of $\mathbb{E}^2$ up to homeomerism derived from 1-homeotoxal, 2-homeotoxal and $\bar{a}$ -uniform tilings, $\bar{a} \leq 4$. Among these there are seven $(1, 3, 1)$ tilings, 11 $(1, 3, 2)$ tilings, five $(1, 3, 3)$ tilings, one $(1, 3, 4)$ tiling, 14 $(2, 3, 1)$ tilings, 33 $(2, 3, 2)$ tilings, 28 $(2, 3, 3)$ tilings, five $(2, 3, 4)$ tilings, eight $(3, 3, 1)$ tilings, 30 $(3, 3, 2)$ tilings, 13 $(3, 3, 3)$ tilings,*

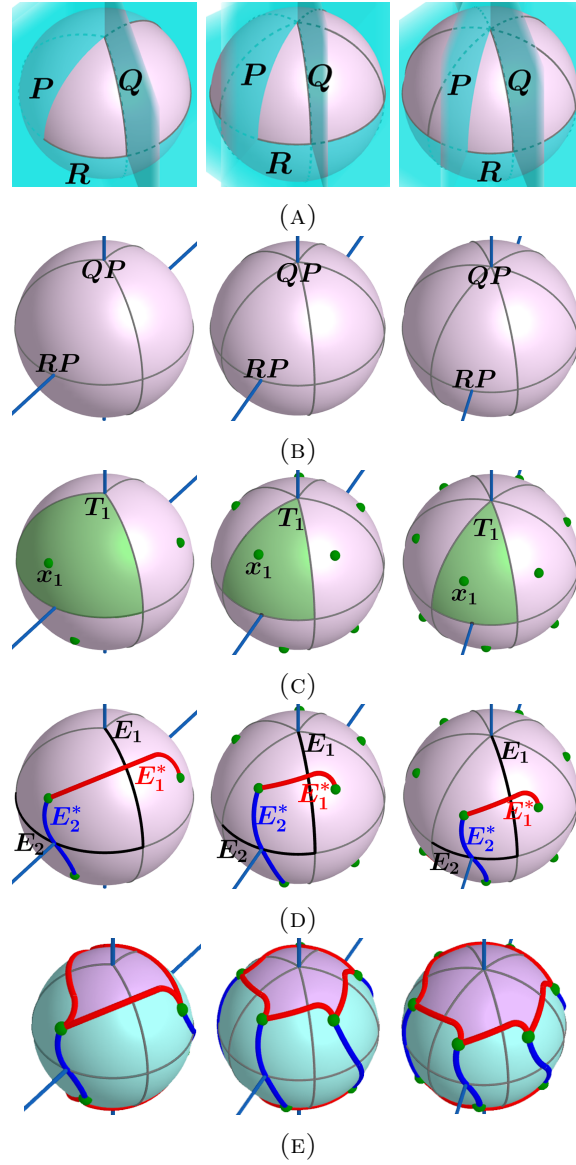


FIGURE 5. Construction of a $(1, 2, 2)$ tiling $\mathcal{T}_n^*$ from $\mathcal{T}_n := \langle 3^2; 4.n \rangle \langle 3^2; 4^2 \rangle$, $n = 3, 5, 6$: (A) $\mathcal{T}_n$ with planes of reflections P , Q , R ; (B) $\mathcal{T}_n$ with axes of rotations QP and RP of subgroup $H_n = \langle QP, RP \rangle \cong n22$; (C) $\mathcal{V} = H_n x_1$; (D) edges E_i^* where $\text{Stab}_{G_n}(E_1^*) \cong C_1$ and $\text{Stab}_{G_n}(E_2^*) = \langle RP \rangle \cong C_2$; and (E) $\mathcal{T}_n^* = H_n E_1^* \cup H_n E_2^*$.

four $(3, 3, 4)$ tilings, one $(4, 3, 1)$ tiling, five $(4, 3, 2)$ tilings, four $(4, 3, 3)$ tilings and one $(4, 3, 4)$ tiling of $\mathbb{E}^2$.

6. (a, b, c) OF $\mathbb{E}^2$ TILINGS WITH CONVEX TILES

In this section we discuss this interesting question: which (a, b, c) tilings of $\mathbb{E}^2$ could be realized by convex tiles only?

Note that every tiling of $\mathbb{E}^2$ by convex tiles is polygonal, where an n -gon ($n \geq 3$) is a polygon with n corners and n sides, each of which is a straight line segment joining two corners [18].

To answer the question, we recall that in the process of construction of an (a, b, c) tiling outlined in Theorem 3.1, we choose an edge E_j^* such that $hE_j^* = E_j^*$ for all $h \in \text{Stab}_H(E_j)$ where there are several possibilities in constructing the edges depending on the group structure of $\text{Stab}_H(E_j)$. Thus, an (a, b, c) tiling can be realized with convex tiles only, that is, the tiling can be constructed using straight edges only, if

$$(6.1) \quad \text{Stab}_H(E_j) \cong D_2 \vee j, \quad E_j \text{ an } H \text{ edge orbit representative.}$$

In the construction of an $(a, 2, c)$ tiling with convex tiles only, one can first check for a given starting tiling $\mathcal{T}$ if each of its edge orbit representatives has a stabilizer group structure D_2 in the symmetry group G of $\mathcal{T}$. Not all edges of a 2-homeotoxal tiling of $\mathbb{E}^2$ satisfy this condition. If $\mathcal{T}$ is a homeotoxal tiling of $\mathbb{E}^2$, the condition is satisfied by the triangle tiling $\langle 3^2; 6^2 \rangle$, square tiling $\langle 4^2; 4^2 \rangle$ and the hexagonal tiling $\langle 6^2; 3^2 \rangle$. The next step is to check for each of these three tilings if all the edge orbit representatives of the tiling have stabilizer D_2 under a subgroup H used to construct an $(a, 2, c)$ tiling. Using Theorem 3.3, the subgroup H satisfying (6.1) is of index 2 in G . It turns out that there is a situation where this happens, and this is when the starting tiling is $\mathcal{T} := \langle 4^2; 4^2 \rangle$ with symmetry group $G = \langle P, Q, R \rangle \cong *442$ and $H = \langle P, R, QPQ, QRQ \rangle \cong *2222$, $[G : H] = 2$. The resulting tiling is the $(1, 2, 1)$ tiling appearing in no. 1 in Table 1. We have the following result.

Proposition 6.1. *There is only one $(a, 2, c)$ tiling of $\mathbb{E}^2$ that can be realized by convex tiles only, the $(1, 2, 1)$ tiling given in no. 1 in Table 1.*

For the case of the $(a, 3, c)$ tilings of $\mathbb{E}^2$, a check on the starting tilings $\langle 3^2; 6^2 \rangle$, $\langle 4^2; 4^2 \rangle$ and $\langle 6^2; 3^2 \rangle$ confirm that not all the edge orbit representatives have stabilizer D_2 under a subgroup H used to construct an $(a, 3, c)$ tiling. Thus, we have not found an $(a, 3, c)$ tiling of $\mathbb{E}^2$ that can be realized by convex tiles only.

7. (a, b, c) TILINGS OF $\mathbb{H}^2$

Hyperbolic (a, b, c) tilings remain scarce in the literature. In this section, we exhibit examples that are obtained from regular and semi-regular tilings. In constructing an (a, b, c) tiling from a regular or semiregular tiling with

symmetry group G , it is helpful to consider Equations (3.1)-(3.3) in finding a subgroup H to generate the desired (a, b, c) tiling.

An example of a $(2, 2, 2)$ tiling $\mathcal{T}_1^*$ is shown in Figure 6A. $\mathcal{T}_1^*$ is obtained from a regular tiling $\mathcal{T}_1 := \langle 4^2; 5^2 \rangle$ by 4-gons with symmetry group $G_1 = \langle P_1, Q_1, R_1 | P_1^2 = Q_1^2 = R_1^2 = (P_1 R_1)^2 = (P_1 Q_1)^4 = (Q_1 R_1)^5 \rangle \cong *542$. An index 6 subgroup $H_1 = \langle Q_1 P_1, R_1 Q_1 R_1 \rangle \cong 4 * 5$ is used to obtain $\mathcal{T}_1^*$ which has symbol $\mathcal{S}_1 := \langle 5^{D_1}.5^{D_1}; 4^{D_1}.4^{C_4} \rangle^{C_1} \langle 5^{D_1}.5^{D_5}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ and symmetry group $G_1^* = H_1 \cong 4 * 5$.

A second example is the $(4, 3, 2)$ tiling $\mathcal{T}_2^*$ shown in Figure 6B obtained from the semiregular tiling $\mathcal{T}_2 = \langle 4.6; 4^2 \rangle$ by regular 4-gons and 6-gons using the subgroup $H_2 = \langle P_2, Q_2, R_2 | P_2^2 = Q_2^2 = R_2^2 = (P_2 R_2)^2 = (P_2 Q_2)^6 = (Q_2 R_2)^4 \rangle \cong *663$. Here $[G_2 : H_2] = 4$ where $G_2 = \langle P_2, Q_2, R_2 | P_2^2 = Q_2^2 = R_2^2 = (P_2 R_2)^2 = (P_2 Q_2)^6 = (Q_2 R_2)^4 \rangle \cong *642$ is the symmetry group of $\mathcal{T}_2$. The resulting tiling $\mathcal{T}_2^*$ has symbol $\mathcal{S}_2 := \langle 4^{D_1}.4^{D_1}; 4^{D_1}.6^{D_6} \rangle^{D_1} \langle 4^{D_1}.4^{D_1}; 4^{D_1}.6^{D_3} \rangle^{C_1} \langle 4^{D_1}.4^{D_1}; 4^{D_1}.6^{D_6} \rangle^{D_1}$ and symmetry group $G_2^* = H_2 \cong *663$.

Finally, we have the $(3, 2, 2)$ tiling $\mathcal{T}_3^*$ shown in Figure 6C. This tiling is obtained from a semiregular tiling $\mathcal{T}_3 := \langle 3.8; 4.4 \rangle$ by 3-gons and 8-gons using a subgroup $H_3 = \langle P_3, R_3, Q_3 | P_3^2 = Q_3^2 = R_3^2 = (P_3 R_3)^2 = (P_3 Q_3)^8 = (Q_3 R_3)^3 \rangle \cong *842$, where $[G_3 : H_3] = 3$, $G_3 = \langle P_3, Q_3, R_3 | P_3^2 = Q_3^2 = R_3^2 = (P_3 R_3)^2 = (P_3 Q_3)^8 = (Q_3 R_3)^3 \rangle \cong *832$ is the symmetry group of $\mathcal{T}_3$. $\mathcal{T}_3^*$ has symbol $\mathcal{S}_3 := \langle 4^{D_1}.4^{D_1}; 3^{D_1}.8^{D_8} \rangle^{D_1} \langle 4^{D_1}.4^{D_2}; 3^{D_1}.8^{D_4} \rangle^{C_1}$ and symmetry group $G_3^* = H_3 \cong *842$.

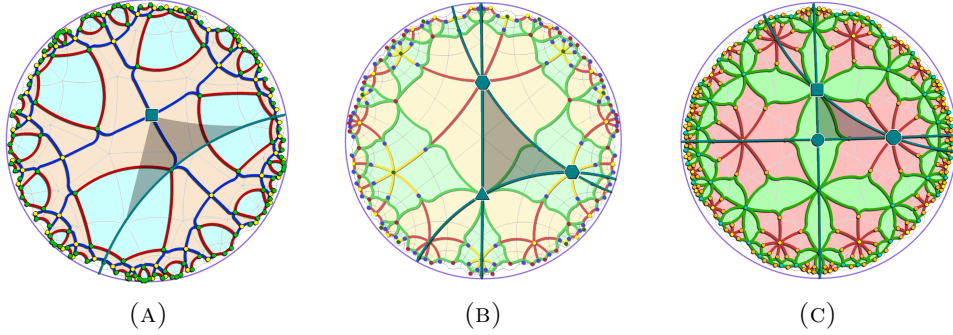


FIGURE 6. (A) $(2, 2, 2)$ tiling; (B) $(4, 3, 2)$ tiling; and (C) $(3, 2, 2)$ tiling of $\mathbb{H}^2$.

8. CONCLUSION AND FUTURE OUTLOOK

In this work, a method is presented of constructing edge-to-edge (a, b, c) tilings of $\mathbb{X}$ ($\mathbb{E}^2$, $\mathbb{H}^2$ or $\mathbb{S}^2$). Parameters are given for the starting tiling with symmetry group G and its subgroups for which, through the given method, an (a, b, c) tiling is obtained.

Applying the given method facilitates the complete enumeration of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ up to homeomerism based on the symmetry groups of

the tilings and their edge transitivity symbols. The approach employed is exhaustive by considering all homeotoxal and 2-homeotoxal tilings as starting tilings, and determining all the possible subgroups (distinct up to conjugacy) of their automorphism groups with the aid of the software GAP that will give rise to $(a, 2, c)$ tilings. From the enumeration in Theorem 4.1, we confirm the existence of 34 normal $(2, 2, 2)$ tilings of $\mathbb{E}^2$ that coincide with the tilings obtained by Dress and Scharlau [12]. In their study, they enumerated the 37 types of minimal, nontransitive, equivariant tilings of $\mathbb{E}^2$, three of which are tilings that have tiles with disconnected intersection. In [6], Delgado-Friedrichs and O’Keeffe enumerated seven $(3, 2, c)$ tilings of $\mathbb{E}^2$ and 12 $(3, 2, c)$ tilings of $\mathbb{S}^2$. These are tilings numbered 94, 98, 99, 100, 101, 102 and 103 in Table 1 and tilings numbered 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81 and 82 in Table 2. Our method points towards nine additional $(3, 2, c)$ tilings of $\mathbb{E}^2$, numbered 89, 90, 91, 92, 93, 95, 96, 97 and 104 in Table 1; four additional $(3, 2, c)$ tilings of $\mathbb{S}^2$, numbered 67, 68, 70 and 72 in Table 2; and one additional infinite sequence of $(3, 2, c)$ tiling of $\mathbb{S}^2$, numbered 15 in Table 3. The list of $(a, 2, c)$ tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ is also in essence, the enumeration of 2-isotoxal tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$. In [12], the numbers of 2-isotoxal tilings of $\mathbb{E}^2$ and $\mathbb{S}^2$ are provided by the authors, but no lists are given.

Our work also gives a list of $(a, 3, c)$ tilings of $\mathbb{E}^2$. Although the enumeration is not complete as there is no known list of 3-homeotoxal tilings, however, we were able to arrive at a substantial list of $(a, 3, c)$ tilings that are obtained from homeotoxal and 2-homeotoxal tilings, as well as from $(\bar{a}, 3, \bar{c})$ tilings consisting of regular polygons, $\bar{a} \leq 4$. In the process, 11 $(4, 3, c)$ tilings are obtained, numbered 160-170 in Table 4 that contribute to the search for $(a, a - 1, c)$ tilings that are not found in the literature. An interested reader may refer to [28] for details on the construction of the tilings in Tables 1-3.

There are other methods that may be employed to construct $(a, 3, c)$ tilings to add to the list. One may start with a uniform tiling having one or two orbits of edges and consider various dissection techniques to arrive at an $(a, 3, c)$ tiling. For example, starting with the $(1, 2, 2)$ uniform tiling $\mathcal{T} := (3.12^2)$ (Figure 7A), dissecting the edges of its 3-gons gives the $(2, 3, 3)$ tiling in Figure 7B; and dissecting the red and blue edges, respectively of its 12-gons, yields the $(2, 3, 3)$ tiling shown in Figure 7C and Figure 7D. These tilings are of different homeomeric types: each having symmetry group type $*632$ and edge transitivity symbols labeled in the caption of Figure 7. It will be interesting as a future study to arrive at a systematic way of carrying out dissections on uniform (a, b, c) tilings to arrive at (a', b', c') tilings, $b' > b$, and completing the list of $(a, 3, c)$ tilings in the process. A dynamic geometry software can be helpful for this purpose (see another related study [4]).

A natural extension of this paper is to consider the three-dimensional analogue of the (a, b, c) tiling, that is, an (a, b, c, d) tiling having a orbits of vertices, b orbits of edges, c orbits of faces and d orbits of cells. One

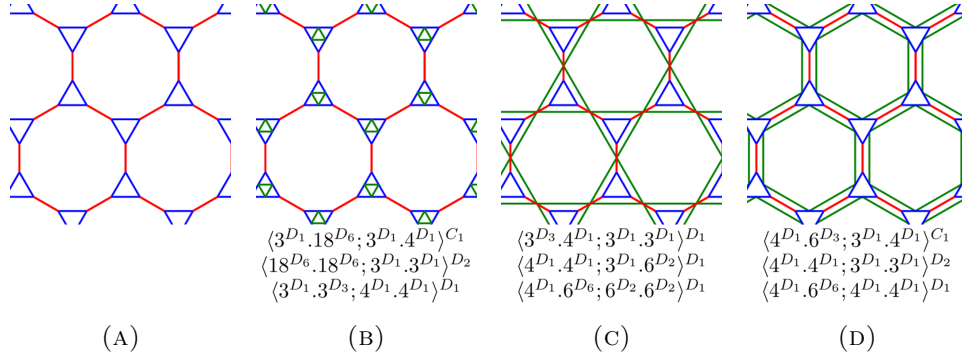


FIGURE 7. (A) The $(1, 2, 2)$ uniform tiling $\mathcal{T} := (3.12^2)$; and (B)-(D) $(2, 3, 3)$ tilings obtained from $\mathcal{T}$ through dissection. The edge transitivity symbols of the $(2, 3, 3)$ tilings are provided.

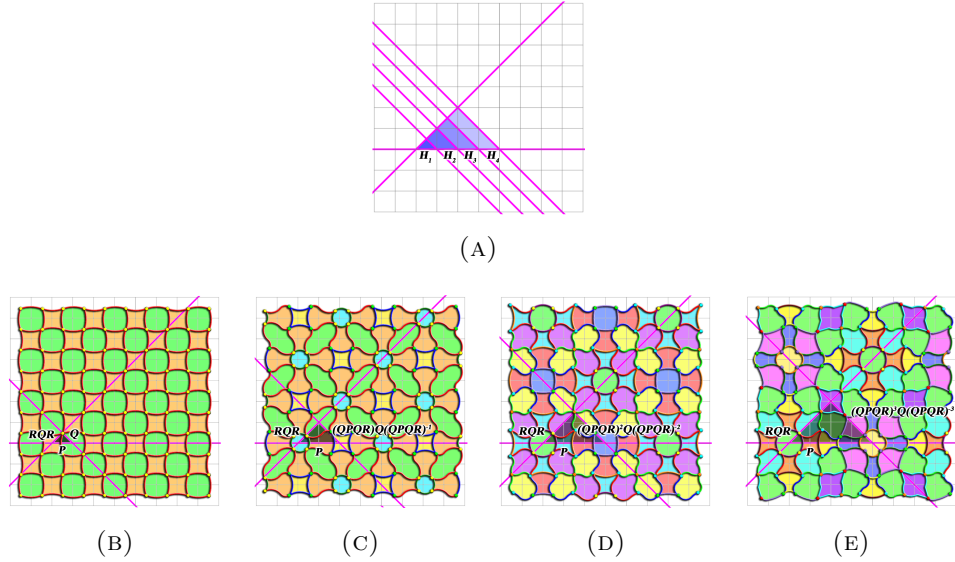


FIGURE 8. (A) The fundamental regions of H_n with axes of reflections of generators of H_n , $n = 1, 2, 3, 4$; (B) $(1, 1, 2)$ tiling obtained using H_1 ; (C) $(2, 3, 4)$ tiling obtained using H_2 ; (D) $(4, 6, 6)$ tiling obtained using H_3 ; and (E) $(6, 10, 9)$ tiling obtained using H_4 .

can study how the method discussed here can be adapted to systematically construct (a, b, c, d) tilings.

Another problem that may be considered is to determine (a, b, c) tilings with a particular symmetry group type, from a given starting tiling with

symmetry group G . The well-known characterization of the subgroup structure of plane crystallographic groups may be helpful in this direction [25, 26]. For example, given a tiling by squares with symmetry group $G = \langle P, Q, R \rangle \cong *442$. Consider the subgroup $H_n = \langle P, RQR, (QPQR)^n Q (QPQR)^{-n} \rangle \cong *442$ of index $2n^2$. By analyzing the number of vertices, edges and tiles in a fundamental region for all n , a conjecture on the type of (a, b, c) tiling can be obtained. H_1 gives the $(1, 1, 2)$ tiling $\mathcal{T}_1^*$ (Figure 8B), H_2 gives the $(2, 3, 4)$ tiling $\mathcal{T}_2^*$ (Figure 8C), H_3 yields the $(4, 6, 6)$ tiling $\mathcal{T}_3^*$ (Figure 8D) and H_4 yields the $(6, 10, 9)$ tiling $\mathcal{T}_4^*$ (Figure 8E). In general, starting with a square tiling with symmetry group G , we obtain a tiling $\mathcal{T}_n^*$ with symmetry group $H_n \cong *442$ and vertex-edge-tile transitivity as $\left(\left\lfloor \frac{(n+1)^2}{4} \right\rfloor, \frac{n^2+n}{2}, \left\lfloor \frac{(n+2)^2}{4} \right\rfloor \right)$.

REFERENCES

- [1] D. P. Chavey, *Periodic tilings and tilings by regular polygons*, Ph.D. Dissertation, University of Wisconsin – Madison, 1984.
- [2] J. H. Conway, *The orbifold notation for surface groups*, In: Groups, Combinatorics and Geometry, London Mathematical Society Lecture Notes Series **165** (1992), 438–447.
- [3] J. H. Conway, H. Burgiel and C. Goodman-Strauss, *The symmetries of things*, A K Peters, Ltd., 2008.
- [4] M. L. A. N. De Las Peñas and E. Taganap, *Discovering new tessellations using dynamic geometry software*, Proceedings of the 22nd Asian Technology Conference in Mathematics (eds. W.C. Yang, D. Meade and Y. Yuan), Mathematics and Technology, LLC, Blacksburg VA, USA. 153–162, 2017.
- [5] O. Delgado-Friedrichs, D. Huson and E. Zamorzaeva, *The classification of 2-isohedral tilings of the plane*, Geom. Dedicata **42** (1992), no. 1, 43–117.
- [6] O. Delgado-Friedrichs and M. O’Keeffe, *Edge-2-transitive trinodal polyhedra and 2-periodic tilings*, Acta Cryst. **A73** (2017), 227–230.
- [7] O. Delgado-Friedrichs, M. O’Keeffe and O. M. Yaghi, *Three-periodic nets and tilings: edge-transitive binodal Structures*, Acta Cryst. **A62** (2006), 350–355.
- [8] ———, *Three-periodic nets and tilings: regular and quasiregular nets*, Acta Cryst. **A59** (2003), 22–27.
- [9] ———, *Three-periodic nets and tilings: semiregular nets*, Acta Cryst. **A59** (2003), 515–525.
- [10] ———, *Three-periodic tilings and nets: face-transitive tilings and edge-transitive nets revisited*, Acta Cryst. **A63** (2007), 344–347.
- [11] A. W. M. Dress and D. Huson, *On tilings of the plane*, Geom. Dedicata **24** (1987), 295–310.
- [12] A. W. M. Dress and R. Scharlau, *The 37 Combinatorial Types of Minimal, Non-transitive, Equivariant Tilings of the Euclidean Plane*, Discrete Math. **60** (1986), 121–138.
- [13] B. Galebach, *n-uniform tilings*, <http://probabilitysports.com/tilings.html>.
- [14] The GAP Group, *GAP — Groups, Algorithms, and Programming, Version 4.11.1*, 2021, <http://www.gap-system.org>.
- [15] B. Grünbaum, H.-D. Löckenhoff, G.C. Shephard and Á. H. Temesvári, *The Enumeration of normal 2-homeohedral tilings*, Geom. Dedicata **19** (1985), no. 2, 109–173.
- [16] B. Grünbaum and G. C. Shephard, *The eighty-one types of isohedral tilings in the plane*, Math. Proc. Cambridge Philos. Soc. **82** (1977), no. 2, 177–196.
- [17] ———, *The 2-homeotoxal tilings of the plane and the 2-sphere*, J. Combin. Theory Ser. B **34** (1983), 113–150.

- [18] ———, *Tilings and Patterns, Second Edition*, Dover Publications, 2016.
- [19] ———, *Isotoxal tilings*, Pacific J. Math. **76** (1978), 407–430.
- [20] ———, *The ninety-one types of isogonal tilings in the plane*, Trans. Amer. Math. Soc. **242** (1978), 335–353.
- [21] ———, *Spherical tilings with transitivity properties*, In: Davis C., Grünbaum B., Sherk F.A. (eds) *The Geometric Vein*, Springer, New York, 1981.
- [22] D. H. Huson, *The generation and classification of tile- k -transitive tilings of the Euclidean plane, the sphere and the hyperbolic plane*, Geom. Dedicata **47** (1993), 269–296.
- [23] O. Krötenheerdt, *Die homogenen Mosaike n -ter Ordnung in der euklidischen Ebene, I*, Wissenschaftliche Zeitschrift der Martin-Luther-Universität Halle-Wittenberg **18** (1969), 273–290.
- [24] Z. Lučić, E. Molnár and M. Stojanović, *The 14 infinite families of isotoxal tilings in the planes of constant curvature*, Period. Math. Hungar. **29** (1994), 177–195.
- [25] T. A. Rapanut, *Subgroups, conjugate subgroups and n -color groups of the seventeen plane crystallographic groups*, Ph.D. Dissertation, University of the Philippines, Diliman, 1988.
- [26] M. Senechal, *Morphisms of crystallographic groups: kernels and images*, J. Math. Phys. **26** (1985), 219–228.
- [27] D. Sommerville, *Semi-regular networks of the plane in absolute geometry*, Trans. Roy. Soc. Edin. **41** (1906), no. 29, 725–747.
- [28] M. Tomenes, *Edge-to-edge tilings with vertex, edge and tile transitivity properties*, Ph.D. Dissertation, Ateneo de Manila University, Quezon City, Philippines, 2021.
- [29] E. Zamorzaeva, *Classification of 2-isohedral tilings of the sphere*, Bul. Acad. Științe Repub. Mold. Mat. **3** (1997), 74–85.
- [30] ———, *Enumeration of 2-isohedral tilings on the sphere*, Visual Mathematics (electronic journal) **6** (2004), no. 1,
- [31] ———, *Non-fundamental 2-isohedral tilings of the sphere*, Bul. Acad. Științe Repub. Mold. Mat. **2** (2008), no. 57, 35–45.

TABLE 1. $(a, 2, c)$ tilings of $\mathbb{E}^2$. The symmetry group in Conway notation and edge transitivity symbol of each tiling are given. Vertices, edges or tiles belonging to one orbit under the symmetry group of the tiling are given the same color. Tilings numbered 1, 2, 3, 4, 5, 6, 7, 47, 48, 49, 50 and 104 are self-dual.

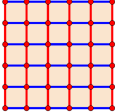
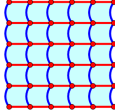
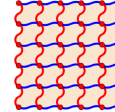
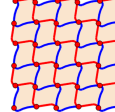
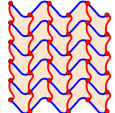
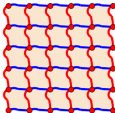
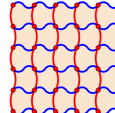
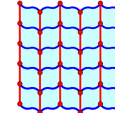
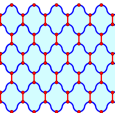
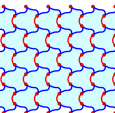
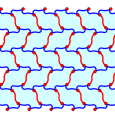
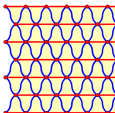
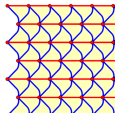
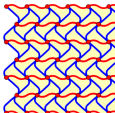
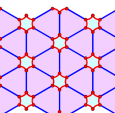
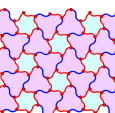
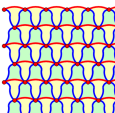
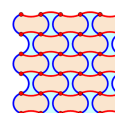
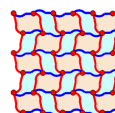
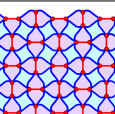
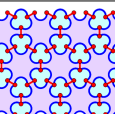
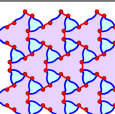
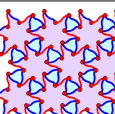
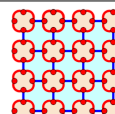
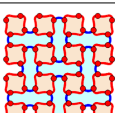
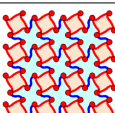
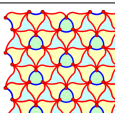
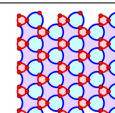
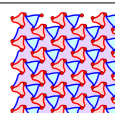
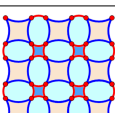
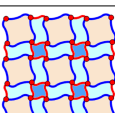
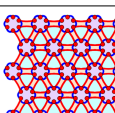
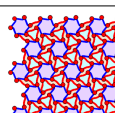
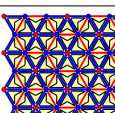
				
1 $(1, 2, 1); *2222;$ $\langle 4^{D_2}.4^{D_2}; 4^{D_2}.4^{D_2} \rangle_{D_2}$ $\langle 4^{D_2}.4^{D_2}; 4^{D_2}.4^{D_2} \rangle_{D_2}$	2 $(1, 2, 1); **;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	3 $(1, 2, 1); 22*;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_2}$	4 $(1, 2, 1); 2222;$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_2}$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_2}$	5 $(1, 2, 1); \times \times;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$
				
6 $(1, 2, 1); \times \times;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	7 $(1, 2, 1); o;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	8 $(1, 2, 1); 22*;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{C_2} \rangle_{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{C_2} \rangle_{D_1}$	9 $(1, 2, 1); 22*;$ $\langle 4^{C_2}.4^{C_2}; 4^{D_1}.4^{D_1} \rangle_{C_2}$ $\langle 4^{C_2}.4^{C_2}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	10 $(1, 2, 1); 2 * 22;$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{C_2}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_2}$
				
11 $(1, 2, 1); * \times;$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	12 $(1, 2, 1); 22 \times;$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_2}$	13 $(1, 2, 1); 2 * 22;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle_{C_2}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle_{D_2}$	14 $(1, 2, 1); * \times;$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$	15 $(1, 2, 1); 22 \times;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_2}$
				
16 $(1, 2, 2); *632;$ $\langle 6^{D_3}.6^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_2}$ $\langle 6^{D_3}.6^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	17 $(1, 2, 2); 632;$ $\langle 6^{C_3}.6^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 6^{C_3}.6^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	18 $(1, 2, 2); * \times;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$	19 $(1, 2, 2); 2 * 22;$ $\langle 4^{D_2}.4^{D_2}; 4^{C_2}.4^{C_2} \rangle_{D_1}$ $\langle 4^{D_2}.4^{D_2}; 4^{C_2}.4^{C_2} \rangle_{D_1}$	20 $(1, 2, 2); 22 \times;$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$
				
21 $(1, 2, 2); 4 * 2;$ $\langle 3^{D_1}.4^{C_4}; 5^{D_1}.5^{D_1} \rangle_{D_2}$ $\langle 3^{D_1}.3^{D_1}; 5^{D_1}.5^{D_1} \rangle_{D_2}$	22 $(1, 2, 2); *632;$ $\langle 3^{D_3}.12^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 12^{D_6}.12^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_2}$	23 $(1, 2, 2); 3 * 3;$ $\langle 3^{C_3}.12^{D_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 12^{D_3}.12^{D_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	24 $(1, 2, 2); 632;$ $\langle 3^{C_3}.12^{C_6}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 12^{C_6}.12^{C_6}; 3^{C_1}.3^{C_1} \rangle_{C_2}$	25 $(1, 2, 2); *442;$ $\langle 8^{D_4}.8^{D_4}; 3^{D_1}.3^{D_1} \rangle_{D_2}$ $\langle 4^{D_4}.8^{D_4}; 3^{D_1}.3^{D_1} \rangle_{D_1}$
				
26 $(1, 2, 2); 4 * 2;$ $\langle 8^{D_2}.8^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 4^{C_4}.8^{D_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	27 $(1, 2, 2); 442;$ $\langle 8^{C_4}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 4^{C_4}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	28 $(1, 2, 3); 3 * 3;$ $\langle 3^{D_1}.3^{D_3}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{C_3}; 6^{D_1}.6^{D_1} \rangle_{C_1}$	29 $(1, 2, 3); *333;$ $\langle 3^{D_3}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 3^{D_3}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	30 $(1, 2, 3); 333;$ $\langle 3^{C_3}.6^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_3}.6^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$
				
31 $(1, 2, 3); *442;$ $\langle 4^{D_2}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	32 $(1, 2, 3); 442;$ $\langle 4^{C_2}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	33 $(1, 2, 3); *632;$ $\langle 3^{D_3}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.6^{D_6}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	34 $(1, 2, 3); 632;$ $\langle 3^{C_3}.4^{C_2}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.6^{C_6}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	35 $(2, 2, 1); *632;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_3}.6^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_3}.6^{D_3} \rangle_{D_2}$

TABLE 1. (continued)

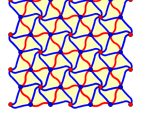
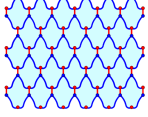
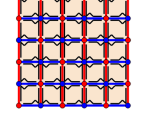
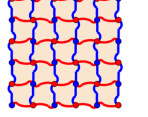
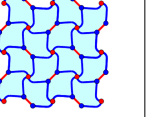
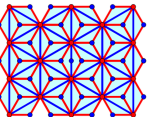
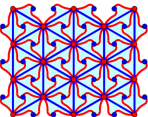
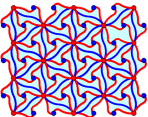
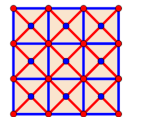
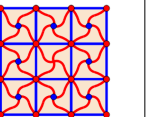
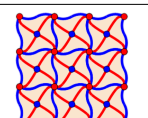
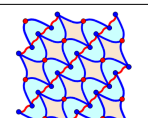
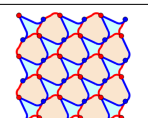
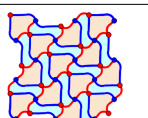
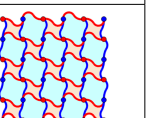
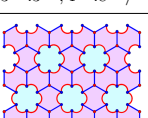
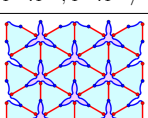
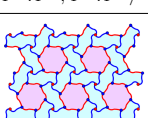
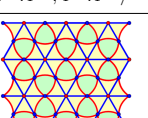
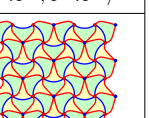
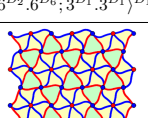
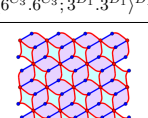
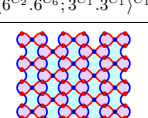
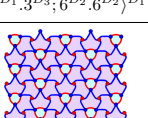
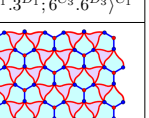
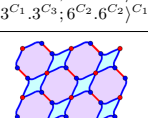
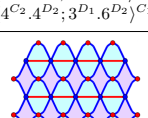
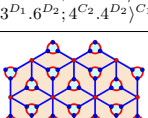
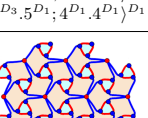
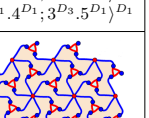
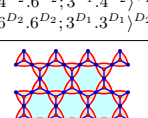
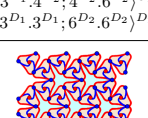
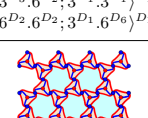
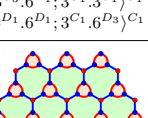
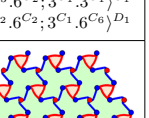
 36 $(2, 2, 1); 632;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_3}.6^{C_6} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 6^{C_3}.6^{C_3} \rangle_{C_2}$	 37 $(2, 2, 1); * \times;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 38 $(2, 2, 1); 2 * 22;$ $\langle 4^{C_2}.4^{C_2}; 4^{D_2}.4^{D_2} \rangle_{D_1}$ $\langle 4^{C_2}.4^{C_2}; 4^{D_2}.4^{D_2} \rangle_{D_1}$	 39 $(2, 2, 1); 22 \times;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle_{C_1}$	 40 $(2, 2, 1); 4 * 2;$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.4^{C_4} \rangle_{C_1}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_2}$
 41 $(2, 2, 1); *632;$ $\langle 3^{D_1}.3^{D_1}; 3^{D_3}.12^{D_6} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 12^{D_6}.12^{D_6} \rangle_{D_2}$	 42 $(2, 2, 1); 3 * 3;$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.12^{D_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 12^{D_3}.12^{D_3} \rangle_{D_1}$	 43 $(2, 2, 1); 632;$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.12^{C_6} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 12^{C_6}.12^{C_6} \rangle_{C_2}$	 44 $(2, 2, 1); *442;$ $\langle 3^{D_1}.3^{D_1}; 8^{D_4}.8^{D_4} \rangle_{D_2}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_4}.8^{D_4} \rangle_{D_1}$	 45 $(2, 2, 1); 4 * 2;$ $\langle 3^{C_1}.3^{C_1}; 8^{D_2}.8^{D_2} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.8^{D_2} \rangle_{C_1}$
 46 $(2, 2, 1); 442;$ $\langle 3^{C_1}.3^{C_1}; 8^{C_4}.8^{C_4} \rangle_{C_2}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.8^{C_4} \rangle_{C_1}$	 47 $(2, 2, 2); 2 * 22;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_2}$	 48 $(2, 2, 2); **;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 49 $(2, 2, 2); 22 *;$ $\langle 4^{C_2}.4^{D_1}; 4^{C_2}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_2}.4^{D_1}; 4^{C_2}.4^{D_1} \rangle_{C_1}$	 50 $(2, 2, 2); 2222;$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_2}.4^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$
 51 $(2, 2, 2); *632;$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_3} \rangle_{D_1}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 52 $(2, 2, 2); 3 * 3;$ $\langle 6^{C_3}.6^{D_3}; 3^{D_1}.3^{D_1} \rangle_{C_1}$ $\langle 6^{C_3}.6^{C_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 53 $(2, 2, 2); 632;$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_3} \rangle_{C_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 54 $(2, 2, 2); *632;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_6} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle_{D_1}$	 55 $(2, 2, 2); 3 * 3;$ $\langle 3^{D_1}.3^{D_1}; 6^{C_3}.6^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{C_3}.6^{C_3} \rangle_{C_1}$
 56 $(2, 2, 2); 632;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_6} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 6^{C_2}.6^{C_2} \rangle_{C_1}$	 57 $(2, 2, 2); 2 * 22;$ $\langle 4^{C_2}.4^{C_2}; 3^{D_1}.6^{D_2} \rangle_{D_1}$ $\langle 4^{C_2}.4^{D_2}; 3^{D_1}.6^{D_2} \rangle_{C_1}$	 58 $(2, 2, 2); 2 * 22;$ $\langle 3^{D_1}.6^{D_2}; 4^{C_2}.4^{C_2} \rangle_{D_1}$ $\langle 3^{D_1}.6^{D_2}; 4^{C_2}.4^{D_2} \rangle_{C_1}$	 59 $(2, 2, 2); 3 * 3;$ $\langle 5^{D_1}.5^{D_1}; 3^{C_3}.4^{D_1} \rangle_{C_1}$ $\langle 3^{D_3}.5^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 60 $(2, 2, 2); 3 * 3;$ $\langle 3^{C_3}.4^{D_1}; 5^{D_1}.5^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.5^{D_1} \rangle_{D_1}$
 61 $(2, 2, 2); *2222;$ $\langle 4^{D_2}.6^{D_2}; 3^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_2}$	 62 $(2, 2, 2); *2222;$ $\langle 3^{D_1}.4^{D_2}; 4^{D_2}.6^{D_2} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle_{D_2}$	 63 $(2, 2, 2); *632;$ $\langle 3^{D_3}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.6^{D_6} \rangle_{D_1}$	 64 $(2, 2, 2); 3 * 3;$ $\langle 3^{C_3}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.6^{D_3} \rangle_{C_1}$	 65 $(2, 2, 2); 632;$ $\langle 3^{C_3}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.6^{C_6} \rangle_{D_1}$
 66 $(2, 2, 2); *632;$ $\langle 3^{D_1}.3^{D_1}; 3^{D_3}.6^{D_2} \rangle_{D_1}$ $\langle 3^{D_1}.6^{D_6}; 6^{D_2}.6^{D_2} \rangle_{D_1}$	 67 $(2, 2, 2); 3 * 3;$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.6^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.6^{D_3}; 6^{D_1}.6^{D_1} \rangle_{C_1}$	 68 $(2, 2, 2); 632;$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.6^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_6}; 6^{C_2}.6^{C_2} \rangle_{D_1}$	 69 $(2, 2, 2); *333;$ $\langle 3^{D_3}.9^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 9^{D_3}.9^{D_3}; 3^{D_1}.3^{D_3} \rangle_{D_1}$	 70 $(2, 2, 2); 333;$ $\langle 3^{C_3}.9^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 9^{C_3}.9^{C_3}; 3^{C_1}.3^{C_3} \rangle_{C_1}$

TABLE 1. (continued)

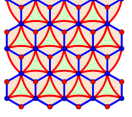
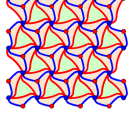
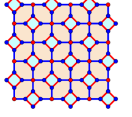
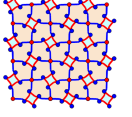
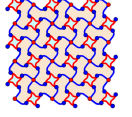
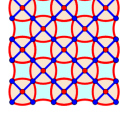
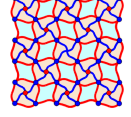
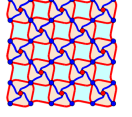
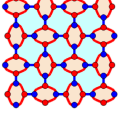
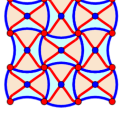
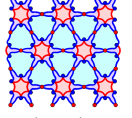
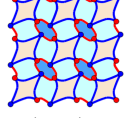
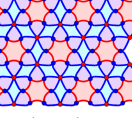
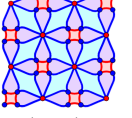
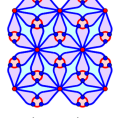
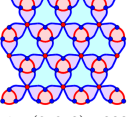
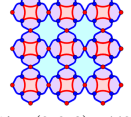
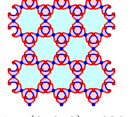
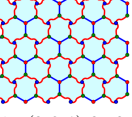
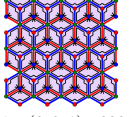
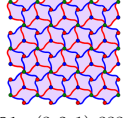
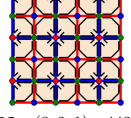
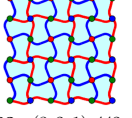
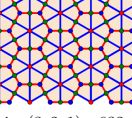
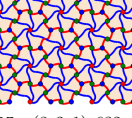
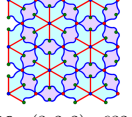
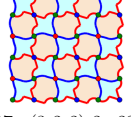
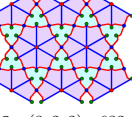
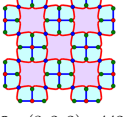
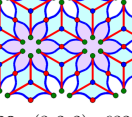
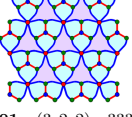
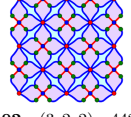
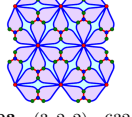
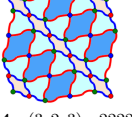
 71 $(2, 2, 2); *333;$ $\langle 3^{D_1}.3^{D_1}; 3^{D_3}.9^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_3}; 9^{D_3}.9^{D_3} \rangle_{D_1}$	 72 $(2, 2, 2); 333;$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.9^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 9^{C_3}.9^{C_3} \rangle_{C_1}$	 73 $(2, 2, 2); *442;$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 4^{D_4}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 74 $(2, 2, 2); 4 * 2;$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{C_4}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 75 $(2, 2, 2); 442;$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.4^{C_4} \rangle_{C_1}$ $\langle 4^{C_4}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$
 76 $(2, 2, 2); *442;$ $\langle 3^{D_1}.4^{D_4}; 6^{D_2}.6^{D_2} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 4^{C_4}.6^{D_1} \rangle_{D_1}$	 77 $(2, 2, 2); 4 * 2;$ $\langle 3^{C_1}.4^{D_2}; 6^{D_1}.6^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.6^{D_1} \rangle_{C_1}$	 78 $(2, 2, 2); 442;$ $\langle 3^{C_1}.4^{D_2}; 6^{C_2}.6^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.6^{C_2} \rangle_{C_1}$	 79 $(2, 2, 2); 4 * 2;$ $\langle 8^{C_4}.8^{C_4}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.8^{C_4}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 80 $(2, 2, 2); 4 * 2;$ $\langle 3^{D_1}.3^{D_1}; 8^{C_4}.8^{C_4} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_2}.8^{C_4} \rangle_{D_1}$
 81 $(2, 2, 3); *632;$ $\langle 3^{D_1}.6^{D_3}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 3^{D_1}.6^{D_6}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 82 $(2, 2, 3); 2 * 22;$ $\langle 4^{C_2}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_2}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 83 $(2, 2, 3); *632;$ $\langle 3^{D_1}.4^{D_2}; 4^{D_1}.6^{D_3} \rangle_{C_1}$ $\langle 3^{D_1}.6^{D_6}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 84 $(2, 2, 3); *442;$ $\langle 3^{D_1}.4^{D_2}; 4^{D_1}.8^{D_4} \rangle_{C_1}$ $\langle 3^{D_1}.4^{D_4}; 4^{D_1}.3^{D_1} \rangle_{D_1}$	 85 $(2, 2, 3); *632;$ $\langle 3^{D_1}.4^{D_2}; 4^{D_1}.12^{D_6} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_3}; 4^{D_1}.4^{D_1} \rangle_{D_1}$
 86 $(2, 2, 3); *333;$ $\langle 3^{D_1}.6^{D_3}; 4^{D_1}.6^{D_3} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_3}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 87 $(2, 2, 3); *442;$ $\langle 3^{D_1}.8^{D_4}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 3^{D_1}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 88 $(2, 2, 3); *632;$ $\langle 3^{D_1}.12^{D_6}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_3}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 89 $(3, 2, 1); 3 * 3;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_3} \rangle_{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{C_3} \rangle_{C_1}$	 90 $(3, 2, 1); *333;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.6^{D_3} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.6^{D_3} \rangle_{D_1}$
 91 $(3, 2, 1); 333;$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.6^{C_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.6^{C_3} \rangle_{C_1}$	 92 $(3, 2, 1); *442;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.4^{D_4} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.4^{D_4} \rangle_{D_1}$	 93 $(3, 2, 1); 442;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_4} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_4} \rangle_{C_1}$	 94 $(3, 2, 1); *632;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_2} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.6^{D_6} \rangle_{D_1}$	 95 $(3, 2, 1); 632;$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.6^{C_6} \rangle_{C_1}$
 96 $(3, 2, 2); *632;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.6^{D_3} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_6} \rangle_{D_1}$	 97 $(3, 2, 2); 2 * 22;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{D_2} \rangle_{C_1}$	 98 $(3, 2, 2); *632;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_6} \rangle_{D_1}$ $\langle 4^{D_1}.6^{D_3}; 3^{D_1}.4^{D_2} \rangle_{C_1}$	 99 $(3, 2, 2); *442;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 4^{D_1}.8^{D_4}; 3^{D_1}.4^{D_2} \rangle_{C_1}$	 100 $(3, 2, 2); *632;$ $\langle 4^{D_1}.12^{D_6}; 3^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle_{D_1}$
 101 $(3, 2, 2); *333;$ $\langle 4^{D_1}.6^{D_3}; 3^{D_1}.6^{D_3} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle_{D_1}$	 102 $(3, 2, 2); *442;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.8^{D_4} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.4^{D_4} \rangle_{D_1}$	 103 $(3, 2, 2); *632;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.12^{D_6} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle_{D_1}$	 104 $(3, 2, 3); *2222;$ $\langle 4^{D_1}.4^{D_2}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.4^{D_2} \rangle_{C_1}$	

TABLE 2. $(a, 2, c)$ tilings of $\mathbb{S}^2$. The symmetry group in Conway notation and edge transitivity symbol of each tiling are given. Vertices, edges or tiles belonging to one orbit under the symmetry group of the tiling are given the same color. Tilings numbered 1 and 2 are self-duals.

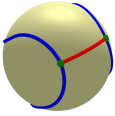
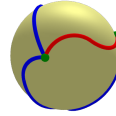
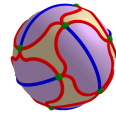
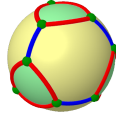
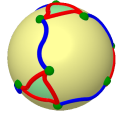
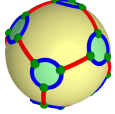
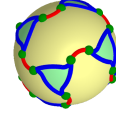
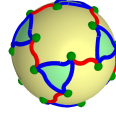
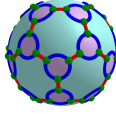
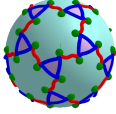
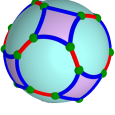
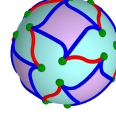
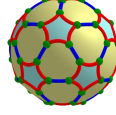
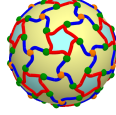
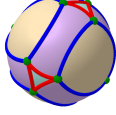
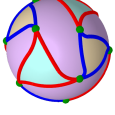
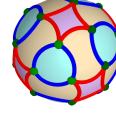
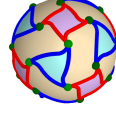
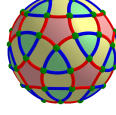
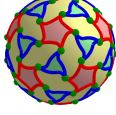
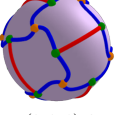
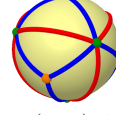
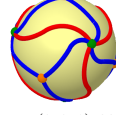
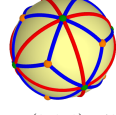
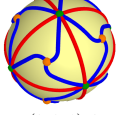
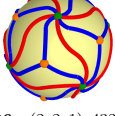
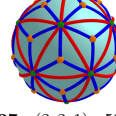
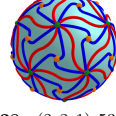
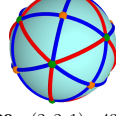
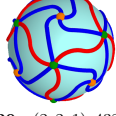
 1 $(1, 2, 1); 2 * 2;$ $\langle 3^{D_1}, 3^{D_1}; 3^{D_1}, 3^{D_1} \rangle_{C_2}$ $\langle 3^{D_1}, 3^{D_1}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$	 2 $(1, 2, 1); 2 \times;$ $\langle 3^{C_1}, 3^{C_1}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$	 3 $(1, 2, 2); 3 * 2;$ $\langle 3^{D_1}, 3^{D_1}; 5^{D_1}, 5^{D_1} \rangle_{D_2}$ $\langle 3^{D_1}, 3^{D_1}; 5^{D_1}, 5^{D_1} \rangle_{C_1}$	 4 $(1, 2, 2); *332;$ $\langle 6^{D_3}, 6^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$ $\langle 3^{D_3}, 6^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_1}$	 5 $(1, 2, 2); 332;$ $\langle 6^{C_3}, 6^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$ $\langle 3^{C_3}, 6^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$
 6 $(1, 2, 2); *432;$ $\langle 8^{D_4}, 3^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_1}$ $\langle 8^{D_4}, 8^{D_4}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$	 7 $(1, 2, 2); 3 * 2;$ $\langle 8^{D_2}, 3^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$ $\langle 8^{D_2}, 8^{D_2}; 3^{C_1}, 3^{C_1} \rangle_{D_1}$	 8 $(1, 2, 2); 432;$ $\langle 8^{C_4}, 3^{D_3}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$ $\langle 8^{C_4}, 8^{C_4}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$	 9 $(1, 2, 2); *532;$ $\langle 10^{D_5}, 3^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_1}$ $\langle 10^{D_5}, 10^{D_5}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$	 10 $(1, 2, 2); 532;$ $\langle 10^{C_5}, 3^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$ $\langle 10^{C_5}, 10^{C_5}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$
 11 $(1, 2, 2); *432;$ $\langle 6^{D_3}, 4^{D_4}; 3^{D_1}, 3^{D_1} \rangle_{D_1}$ $\langle 6^{D_3}, 6^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$	 12 $(1, 2, 2); 432;$ $\langle 6^{C_3}, 4^{C_4}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$ $\langle 6^{C_3}, 6^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$	 13 $(1, 2, 2); *532;$ $\langle 6^{D_3}, 6^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$ $\langle 5^{D_5}, 6^{D_3}; 3^{D_1}, 3^{D_1} \rangle_{D_1}$	 14 $(1, 2, 2); 532;$ $\langle 6^{C_3}, 6^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_2}$ $\langle 5^{C_5}, 6^{C_3}; 3^{C_1}, 3^{C_1} \rangle_{C_1}$	 15 $(1, 2, 3); *332;$ $\langle 3^{D_3}, 4^{D_2}; 4^{D_1}, 4^{D_1} \rangle_{D_1}$ $\langle 3^{D_3}, 4^{D_2}; 4^{D_1}, 4^{D_1} \rangle_{D_2}$
 16 $(1, 2, 3); 332;$ $\langle 3^{C_3}, 4^{C_2}; 4^{C_1}, 4^{C_1} \rangle_{C_1}$ $\langle 3^{C_3}, 4^{C_2}; 4^{C_1}, 4^{C_1} \rangle_{C_2}$	 17 $(1, 2, 3); *432;$ $\langle 3^{D_3}, 4^{D_2}; 4^{D_1}, 4^{D_1} \rangle_{D_1}$ $\langle 4^{D_4}, 4^{D_2}; 4^{D_1}, 4^{D_1} \rangle_{D_2}$	 18 $(1, 2, 3); 432;$ $\langle 3^{C_3}, 4^{C_2}; 4^{C_1}, 4^{C_1} \rangle_{C_1}$ $\langle 4^{C_4}, 4^{C_2}; 4^{C_1}, 4^{C_1} \rangle_{C_2}$	 19 $(1, 2, 3); *532;$ $\langle 3^{D_3}, 4^{D_2}; 4^{D_1}, 4^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}, 5^{D_5}; 4^{D_1}, 4^{D_1} \rangle_{D_2}$	 20 $(1, 2, 3); 532;$ $\langle 3^{C_3}, 4^{C_2}; 4^{C_1}, 4^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}, 5^{C_5}; 4^{C_1}, 4^{C_1} \rangle_{C_2}$
 21 $(2, 2, 1); 3 * 2;$ $\langle 5^{D_1}, 5^{D_1}; 3^{D_1}, 3^{D_1} \rangle_{C_1}$ $\langle 5^{D_1}, 5^{D_1}; 3^{D_1}, 3^{D_1} \rangle_{D_2}$	 22 $(2, 2, 1); *332;$ $\langle 3^{D_1}, 3^{D_1}; 3^{D_3}, 6^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}, 3^{D_1}; 6^{D_3}, 6^{D_3} \rangle_{D_2}$	 23 $(2, 2, 1); 332;$ $\langle 3^{C_1}, 3^{C_1}; 3^{C_3}, 6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 6^{C_3}, 6^{C_3} \rangle_{C_2}$	 24 $(2, 2, 1); *432;$ $\langle 3^{D_1}, 3^{D_1}; 8^{D_4}, 3^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}, 3^{D_1}; 8^{D_4}, 8^{D_4} \rangle_{D_2}$	 25 $(2, 2, 1); 3 * 2;$ $\langle 3^{C_1}, 3^{C_1}; 8^{D_2}, 3^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 8^{D_2}, 8^{D_2} \rangle_{C_2}$
 26 $(2, 2, 1); 432;$ $\langle 3^{C_1}, 3^{C_1}; 8^{C_4}, 3^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 8^{C_4}, 8^{C_4} \rangle_{C_2}$	 27 $(2, 2, 1); *532;$ $\langle 3^{D_1}, 3^{D_1}; 10^{D_5}, 3^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}, 3^{D_1}; 10^{D_5}, 10^{D_5} \rangle_{D_2}$	 28 $(2, 2, 1); 532;$ $\langle 3^{C_1}, 3^{C_1}; 10^{C_5}, 3^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 10^{C_5}, 10^{C_5} \rangle_{C_2}$	 29 $(2, 2, 1); *432;$ $\langle 3^{D_1}, 3^{D_1}; 6^{D_3}, 4^{D_4} \rangle_{D_1}$ $\langle 3^{D_1}, 3^{D_1}; 6^{D_3}, 6^{D_3} \rangle_{D_2}$	 30 $(2, 2, 1); 432;$ $\langle 3^{C_1}, 3^{C_1}; 6^{C_3}, 4^{C_4} \rangle_{C_1}$ $\langle 3^{C_1}, 3^{C_1}; 6^{C_3}, 6^{C_3} \rangle_{C_2}$

TABLE 2. (continued)

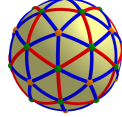
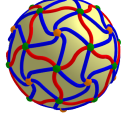
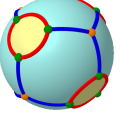
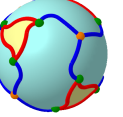
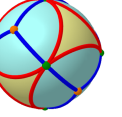
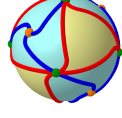
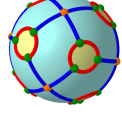
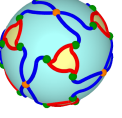
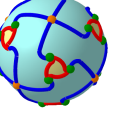
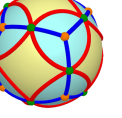
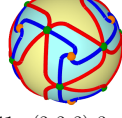
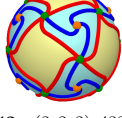
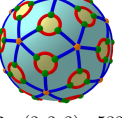
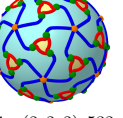
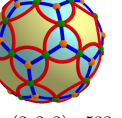
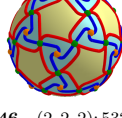
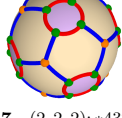
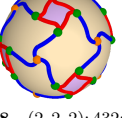
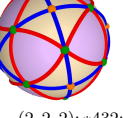
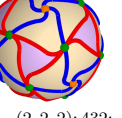
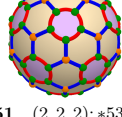
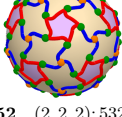
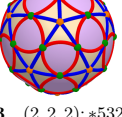
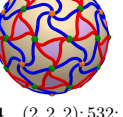
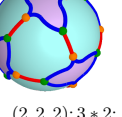
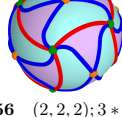
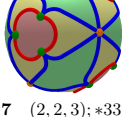
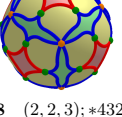
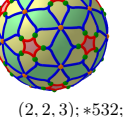
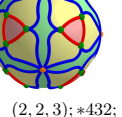
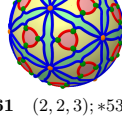
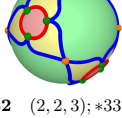
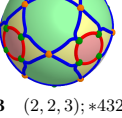
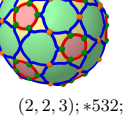
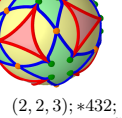
 31 (2, 2, 1); *532; $\langle 3^{D_1}.3^{D_1}.5^{D_5}.6^{D_3} \rangle^{D_1}$ $\langle 3^{D_1}.3^{D_1}.6^{D_3}.6^{D_3} \rangle^{D_2}$	 32 (2, 2, 1); 532; $\langle 3^{C_1}.3^{C_1}.5^{C_5}.6^{C_3} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}.6^{C_3}.6^{C_3} \rangle^{C_2}$	 33 (2, 2, 2); *332; $\langle 6^{D_2}.6^{D_2}.3^{D_1}.3^{D_3} \rangle^{D_1}$ $\langle 3^{D_3}.6^{D_2}.3^{D_1}.3^{D_1} \rangle^{D_1}$	 34 (2, 2, 2); 332; $\langle 6^{C_2}.6^{C_2}.3^{C_1}.3^{C_3} \rangle^{C_1}$ $\langle 3^{C_3}.6^{C_2}.3^{C_1}.3^{C_1} \rangle^{C_1}$	 35 (2, 2, 2); *332; $\langle 3^{D_1}.3^{D_1}.3^{D_3}.6^{D_2} \rangle^{D_1}$ $\langle 3^{D_1}.3^{D_3}.6^{D_2}.6^{D_2} \rangle^{D_1}$
 36 (2, 2, 2); 332; $\langle 3^{C_1}.3^{C_1}.3^{C_3}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_3}.6^{C_2}.6^{C_2} \rangle^{C_1}$	 37 (2, 2, 2); *432; $\langle 6^{D_2}.6^{D_2}.3^{D_1}.4^{D_4} \rangle^{D_1}$ $\langle 3^{D_3}.6^{D_2}.3^{D_1}.3^{D_1} \rangle^{D_1}$	 38 (2, 2, 2); 3 * 2; $\langle 6^{D_1}.6^{D_1}.3^{C_1}.4^{D_4} \rangle^{C_1}$ $\langle 3^{C_3}.6^{D_1}.3^{C_1}.3^{C_1} \rangle^{C_1}$	 39 (2, 2, 2); 432; $\langle 6^{C_2}.6^{C_2}.3^{C_1}.4^{C_4} \rangle^{D_1}$ $\langle 3^{C_3}.6^{C_2}.3^{C_1}.3^{C_1} \rangle^{D_1}$	 40 (2, 2, 2); *432; $\langle 3^{D_1}.3^{D_1}.3^{D_3}.6^{D_2} \rangle^{D_1}$ $\langle 3^{D_1}.4^{D_4}.6^{D_2}.6^{D_2} \rangle^{D_1}$
 41 (2, 2, 2); 3 * 2; $\langle 3^{C_1}.3^{C_1}.3^{C_3}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.4^{D_4}.6^{D_1}.6^{D_1} \rangle^{C_1}$	 42 (2, 2, 2); 432; $\langle 3^{C_1}.3^{C_1}.3^{C_3}.6^{C_2} \rangle^{D_1}$ $\langle 3^{C_1}.4^{C_4}.6^{C_2}.6^{C_2} \rangle^{D_1}$	 43 (2, 2, 2); *532; $\langle 6^{D_2}.6^{D_2}.3^{D_1}.5^{D_5} \rangle^{D_1}$ $\langle 3^{D_3}.6^{D_2}.3^{D_1}.3^{D_1} \rangle^{D_1}$	 44 (2, 2, 2); 532; $\langle 6^{C_2}.6^{C_2}.3^{C_1}.5^{C_5} \rangle^{C_1}$ $\langle 3^{C_3}.6^{C_2}.3^{C_1}.3^{C_1} \rangle^{C_1}$	 45 (2, 2, 2); *532; $\langle 3^{D_1}.3^{D_1}.3^{D_3}.6^{D_2} \rangle^{D_1}$ $\langle 3^{D_1}.5^{D_5}.6^{D_2}.6^{D_2} \rangle^{D_1}$
 46 (2, 2, 2); 532; $\langle 3^{C_1}.3^{C_1}.3^{C_3}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.5^{C_5}.6^{C_2}.6^{C_2} \rangle^{C_1}$	 47 (2, 2, 2); *432; $\langle 6^{D_2}.6^{D_2}.3^{D_1}.3^{D_3} \rangle^{D_1}$ $\langle 4^{D_4}.6^{D_2}.3^{D_1}.3^{D_1} \rangle^{D_1}$	 48 (2, 2, 2); 432; $\langle 6^{C_2}.6^{C_2}.3^{C_1}.3^{C_3} \rangle^{C_1}$ $\langle 4^{C_4}.6^{C_2}.3^{C_1}.3^{C_1} \rangle^{C_1}$	 49 (2, 2, 2); *432; $\langle 3^{D_1}.3^{D_1}.4^{D_4}.6^{D_2} \rangle^{D_1}$ $\langle 3^{D_1}.3^{D_3}.6^{D_2}.6^{D_2} \rangle^{D_1}$	 50 (2, 2, 2); 432; $\langle 3^{C_1}.3^{C_1}.4^{C_4}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_3}.6^{C_2}.6^{C_2} \rangle^{C_1}$
 51 (2, 2, 2); *532; $\langle 6^{D_2}.6^{D_2}.3^{D_1}.3^{D_3} \rangle^{D_1}$ $\langle 5^{D_5}.6^{D_2}.3^{D_1}.3^{D_1} \rangle^{D_1}$	 52 (2, 2, 2); 532; $\langle 6^{C_2}.6^{C_2}.3^{C_1}.3^{C_3} \rangle^{C_1}$ $\langle 5^{C_5}.6^{C_2}.3^{C_1}.3^{C_1} \rangle^{C_1}$	 53 (2, 2, 2); *532; $\langle 3^{D_1}.3^{D_1}.5^{D_5}.6^{D_2} \rangle^{D_1}$ $\langle 3^{D_1}.3^{D_3}.6^{D_2}.6^{D_2} \rangle^{D_1}$	 54 (2, 2, 2); 532; $\langle 3^{C_1}.3^{C_1}.5^{C_5}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_3}.6^{C_2}.6^{C_2} \rangle^{C_1}$	 55 (2, 2, 2); 3 * 2; $\langle 6^{C_3}.4^{D_2}.3^{D_1}.3^{D_1} \rangle^{C_1}$ $\langle 6^{C_3}.6^{C_3}.3^{D_1}.3^{D_1} \rangle^{D_1}$
 56 (2, 2, 2); 3 * 2; $\langle 3^{D_1}.3^{D_1}.6^{C_3}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_1}.6^{C_3}.6^{C_3} \rangle^{D_1}$	 57 (2, 2, 3); *332; $\langle 3^{D_1}.4^{D_2}.4^{D_1}.6^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 58 (2, 2, 3); *432; $\langle 3^{D_1}.4^{D_2}.4^{D_1}.6^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.4^{D_4}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 59 (2, 2, 3); *532; $\langle 3^{D_1}.5^{D_5}.4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 3^{D_1}.4^{D_2}.4^{D_1}.6^{D_2} \rangle^{C_1}$	 60 (2, 2, 3); *432; $\langle 3^{D_1}.4^{D_2}.4^{D_1}.8^{D_4} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}.4^{D_1}.4^{D_1} \rangle^{D_1}$
 61 (2, 2, 3); *532; $\langle 3^{D_1}.4^{D_2}.4^{D_1}.10^{D_5} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 62 (2, 2, 3); *332; $\langle 3^{D_1}.6^{D_3}.4^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 63 (2, 2, 3); *432; $\langle 3^{D_1}.6^{D_3}.4^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.4^{D_4}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 64 (2, 2, 3); *532; $\langle 3^{D_1}.6^{D_3}.4^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.5^{D_5}.4^{D_1}.4^{D_1} \rangle^{D_1}$	 65 (2, 2, 3); *432; $\langle 3^{D_1}.8^{D_4}.4^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}.4^{D_1}.4^{D_1} \rangle^{D_1}$

TABLE 2. (continued)

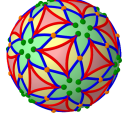
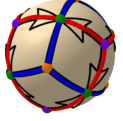
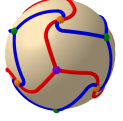
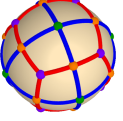
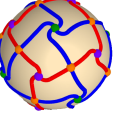
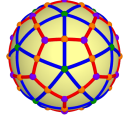
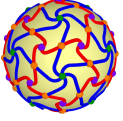
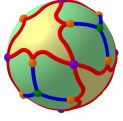
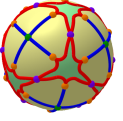
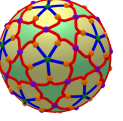
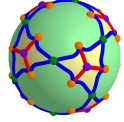
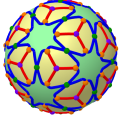
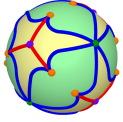
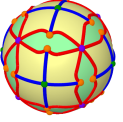
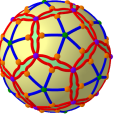
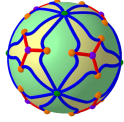
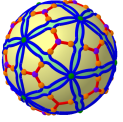
 66 $(2, 2, 3); *532;$ $\langle 3^{D_1}.10^{D_5}; 4^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 3^{D_1}.3^{D_3}; 4^{D_1}.4^{D_2} \rangle^{D_1}$	 67 $(3, 2, 1); *322;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_2} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_2} \rangle^{D_1}$	 68 $(3, 2, 1); 332;$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{C_2} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{C_2} \rangle^{C_1}$	 69 $(3, 2, 1); *432;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_4}.4^{D_2} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_2} \rangle^{D_1}$	 70 $(3, 2, 1); 432;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_4}.4^{C_2} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{C_2} \rangle^{C_1}$
 71 $(3, 2, 1); *532;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.5^{D_5} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_2} \rangle^{D_1}$	 72 $(3, 2, 1); 532;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.5^{C_5} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{C_2} \rangle^{C_1}$	 73 $(3, 2, 2); *332;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$ $\langle 4^{D_1}.6^{D_3}; 3^{D_1}.4^{D_2} \rangle^{C_1}$	 74 $(3, 2, 2); *432;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.4^{D_4} \rangle^{D_1}$ $\langle 4^{D_1}.6^{D_3}; 3^{D_1}.4^{D_2} \rangle^{C_1}$	 75 $(3, 2, 2); *532;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.5^{D_5} \rangle^{D_1}$ $\langle 4^{D_1}.6^{D_3}; 3^{D_1}.4^{D_2} \rangle^{C_1}$
 76 $(3, 2, 2); *432;$ $\langle 4^{D_1}.8^{D_4}; 3^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$	 77 $(3, 2, 2); *532;$ $\langle 4^{D_1}.10^{D_5}; 3^{D_1}.4^{D_2} \rangle^{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$	 78 $(3, 2, 2); *332;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.6^{D_3} \rangle^{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$	 79 $(3, 2, 2); *432;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.4^{D_4} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.6^{D_3} \rangle^{C_1}$	 80 $(3, 2, 2); *532;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.5^{D_5} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.6^{D_3} \rangle^{C_1}$
 81 $(3, 2, 2); *432;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.8^{D_4} \rangle^{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$	 82 $(3, 2, 2); *532;$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.10^{D_5} \rangle^{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_3} \rangle^{D_1}$			

TABLE 3. Infinite Sequences of $(a, 2, c)$ tilings of $\mathbb{S}^2$. The symmetry group in Conway notation and edge transitivity symbol of each tiling are given. Vertices, edges or tiles belonging to one orbit under the symmetry group of the tiling are given the same color. Tilings numbered 11 and 12 are self-duals.

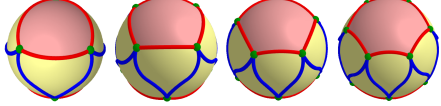
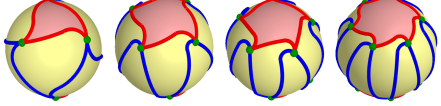
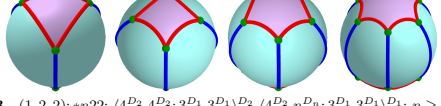
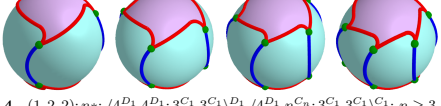
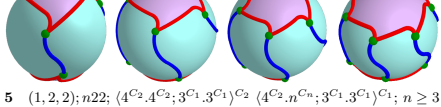
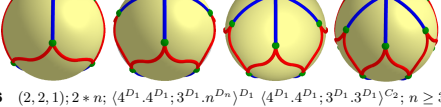
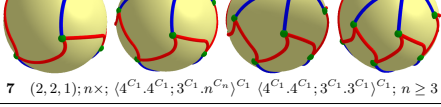
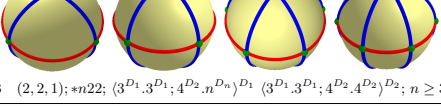
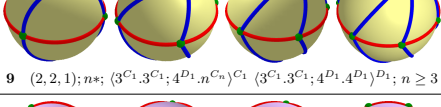
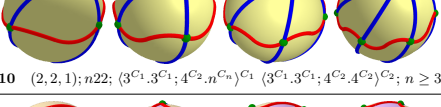
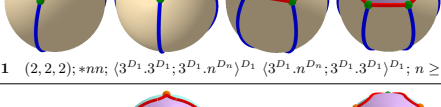
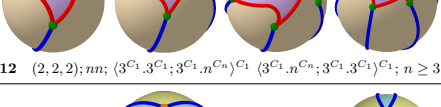
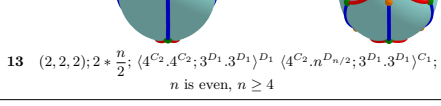
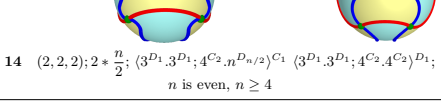
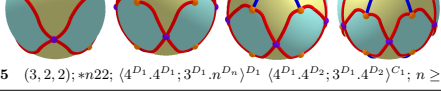
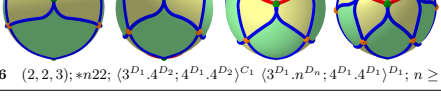
 1 $(1, 2, 2); 2 * n; \langle 3^{D_1}, 3^{D_1}; 4^{D_1}, 4^{D_1} \rangle^{C_2} \langle 3^{D_1}, n^{D_n}; 4^{D_1}, 4^{D_1} \rangle^{D_1}; n \geq 3$	 2 $(1, 2, 2); n \times; \langle 3^{C_1}, 3^{C_1}; 4^{C_1}, 4^{C_1} \rangle^{C_1} \langle 3^{C_1}, n^{C_n}; 4^{C_1}, 4^{C_1} \rangle^{C_1}; n \geq 3$
 3 $(1, 2, 2); *n22; \langle 4^{D_2}, 4^{D_2}; 3^{D_1}, 3^{D_1} \rangle^{D_2} \langle 4^{D_2}, n^{D_n}; 3^{D_1}, 3^{D_1} \rangle^{D_1}; n \geq 3$	 4 $(1, 2, 2); n*; \langle 4^{D_1}, 4^{D_1}; 3^{C_1}, 3^{C_1} \rangle^{D_1} \langle 4^{D_1}, n^{C_n}; 3^{C_1}, 3^{C_1} \rangle^{C_1}; n \geq 3$
 5 $(1, 2, 2); n22; \langle 4^{C_2}, 4^{C_2}; 3^{C_1}, 3^{C_1} \rangle^{C_2} \langle 4^{C_2}, n^{C_n}; 3^{C_1}, 3^{C_1} \rangle^{C_1}; n \geq 3$	 6 $(2, 2, 1); 2 * n; \langle 4^{D_1}, 4^{D_1}; 3^{D_1}, n^{D_n} \rangle^{D_1} \langle 4^{D_1}, 4^{D_1}; 3^{D_1}, 3^{D_1} \rangle^{C_2}; n \geq 3$
 7 $(2, 2, 1); n \times; \langle 4^{C_1}, 4^{C_1}; 3^{C_1}, n^{C_n} \rangle^{C_1} \langle 4^{C_1}, 4^{C_1}; 3^{C_1}, 3^{C_1} \rangle^{C_1}; n \geq 3$	 8 $(2, 2, 1); *n22; \langle 3^{D_1}, 3^{D_1}; 4^{D_2}, n^{D_n} \rangle^{D_1} \langle 3^{D_1}, 3^{D_1}; 4^{D_2}, 4^{D_2} \rangle^{D_2}; n \geq 3$
 9 $(2, 2, 1); n*; \langle 3^{C_1}, 3^{C_1}; 4^{D_1}, n^{C_n} \rangle^{C_1} \langle 3^{C_1}, 3^{C_1}; 4^{D_1}, 4^{D_1} \rangle^{D_1}; n \geq 3$	 10 $(2, 2, 1); n22; \langle 3^{C_1}, 3^{C_1}; 4^{C_2}, n^{C_n} \rangle^{C_1} \langle 3^{C_1}, 3^{C_1}; 4^{C_2}, 4^{C_2} \rangle^{C_2}; n \geq 3$
 11 $(2, 2, 2); *nm; \langle 3^{D_1}, 3^{D_1}; 3^{D_1}, n^{D_n} \rangle^{D_1} \langle 3^{D_1}, n^{D_n}; 3^{D_1}, 3^{D_1} \rangle^{D_1}; n \geq 3$	 12 $(2, 2, 2); nm; \langle 3^{C_1}, 3^{C_1}; 3^{C_1}, n^{C_n} \rangle^{C_1} \langle 3^{C_1}, n^{C_n}; 3^{C_1}, 3^{C_1} \rangle^{C_1}; n \geq 3$
 13 $(2, 2, 2); 2 * \frac{n}{2}; \langle 4^{C_2}, 4^{C_2}; 3^{D_1}, 3^{D_1} \rangle^{D_1} \langle 4^{C_2}, n^{D_{n/2}}; 3^{D_1}, 3^{D_1} \rangle^{C_1};$ $n \text{ is even, } n \geq 4$	 14 $(2, 2, 2); 2 * \frac{n}{2}; \langle 3^{D_1}, 3^{D_1}; 4^{C_2}, n^{D_{n/2}} \rangle^{C_1} \langle 3^{D_1}, 3^{D_1}; 4^{C_2}, 4^{C_2} \rangle^{D_1};$ $n \text{ is even, } n \geq 4$
 15 $(3, 2, 2); *n22; \langle 4^{D_1}, 4^{D_1}; 3^{D_1}, n^{D_n} \rangle^{D_1} \langle 4^{D_1}, 4^{D_2}; 3^{D_1}, 4^{D_2} \rangle^{C_1}; n \geq 3$	 16 $(2, 2, 3); *n22; \langle 3^{D_1}, 4^{D_2}; 4^{D_1}, 4^{D_2} \rangle^{C_1} \langle 3^{D_1}, n^{D_n}; 4^{D_1}, 4^{D_1} \rangle^{D_1}; n \geq 3$

TABLE 4. $(a, 3, c)$ tilings of $\mathbb{E}^2$. The symmetry group in Conway notation and edge transitivity symbol of each tiling are given. Vertices, edges or tiles belonging to one orbit under the symmetry group of the tiling are given the same color. Tilings numbered 1, 2, 3, 39, 40, 41, 42, 143, 144 and 170 are self-duals.

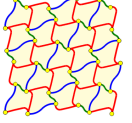
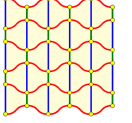
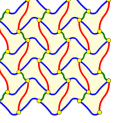
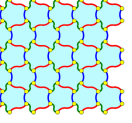
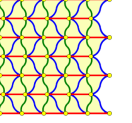
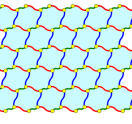
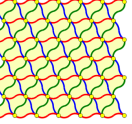
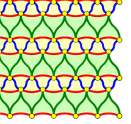
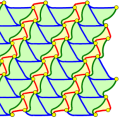
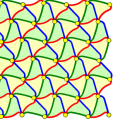
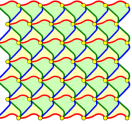
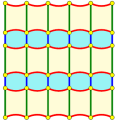
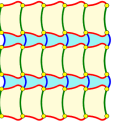
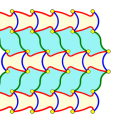
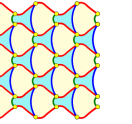
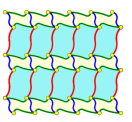
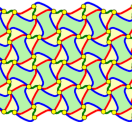
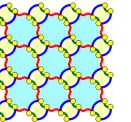
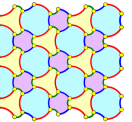
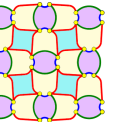
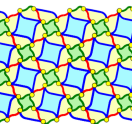
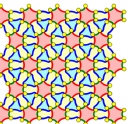
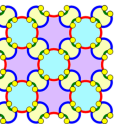
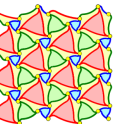
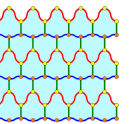
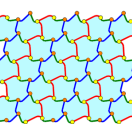
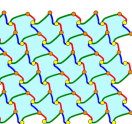
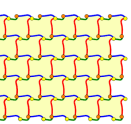
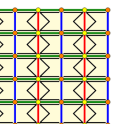
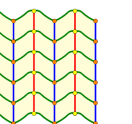
 1 $(1, 3, 1); 22\times;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_2}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_2}$	 2 $(1, 3, 1); 2 * 22;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_2}$	 3 $(1, 3, 1); 22\times;$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_2}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_2}$	 4 $(1, 3, 1); 22\times;$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_2}$	 5 $(1, 3, 1); 22\times;$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{C_2}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{C_2}$
 6 $(1, 3, 1); 2222;$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_2}$	 7 $(1, 3, 1); 2222;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_2}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_2}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_2}$	 8 $(1, 3, 2); 22\times;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{C_2}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{C_2}$	 9 $(1, 3, 2); \times \times;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$	 10 $(1, 3, 2); \times \times;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$
 11 $(1, 3, 2); \circ;$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_1}.6^{C_1} \rangle_{C_1}$	 12 $(1, 3, 2); *2222;$ $\langle 4^{D_2}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_2}$ $\langle 4^{D_2}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_2}$	 13 $(1, 3, 2); **;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$	 14 $(1, 3, 2); 22\times;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_2}$	 15 $(1, 3, 2); **;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$
 16 $(1, 3, 2); 2222;$ $\langle 4^{C_2}.4^{C_2}; 4^{C_1}.4^{C_1} \rangle_{C_2}$ $\langle 4^{C_2}.4^{C_2}; 4^{C_1}.4^{C_1} \rangle_{C_2}$ $\langle 4^{C_2}.4^{C_2}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	 17 $(1, 3, 2); 22\times;$ $\langle 3^{C_1}.4^{C_2}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_2}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_2}; 5^{C_1}.5^{C_1} \rangle_{C_2}$	 18 $(1, 3, 2); 2 * 22;$ $\langle 4^{D_2}.8^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 8^{D_2}.8^{D_2}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 4^{D_2}.8^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	 19 $(1, 3, 3); *333;$ $\langle 6^{D_3}.6^{D_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 6^{D_3}.6^{D_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 6^{D_3}.6^{D_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	 20 $(1, 3, 3); 4 * 2;$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{D_1}$
 21 $(1, 3, 3); 442;$ $\langle 3^{C_1}.4^{C_4}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 5^{C_1}.5^{C_1} \rangle_{C_1}$	 22 $(1, 3, 3); 3 * 3;$ $\langle 3^{C_3}.4^{D_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 4^{D_1}.6^{D_3}; 4^{C_1}.4^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.6^{D_3}; 4^{C_1}.4^{C_1} \rangle_{D_1}$	 23 $(1, 3, 3); *442;$ $\langle 4^{D_2}.8^{D_4}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 8^{D_4}.8^{D_4}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 4^{D_2}.8^{D_4}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	 24 $(1, 3, 4); 333;$ $\langle 3^{C_1}.3^{C_3}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 6^{C_1}.6^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 6^{C_1}.6^{C_1} \rangle_{C_1}$	 25 $(2, 3, 1); 22\times;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{C_2}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{C_2}$
 26 $(2, 3, 1); \times \times \times;$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 27 $(2, 3, 1); \times \times \times;$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 28 $(2, 3, 1); \circ;$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 29 $(2, 3, 1); *2222;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.4^{D_2} \rangle_{D_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.4^{D_2} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_2}.4^{D_2} \rangle_{D_2}$	 30 $(2, 3, 1); **;$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$

TABLE 4. (continued)

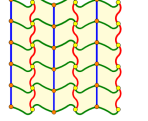
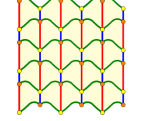
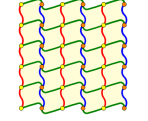
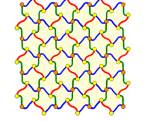
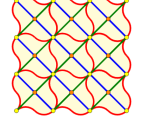
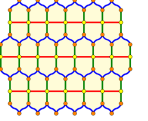
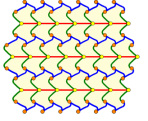
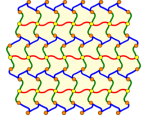
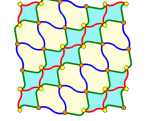
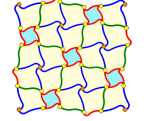
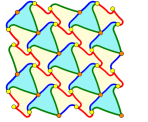
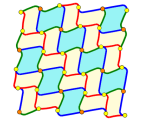
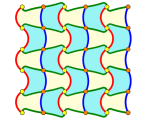
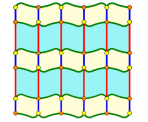
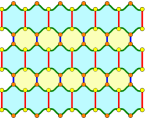
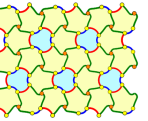
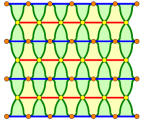
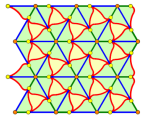
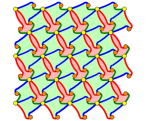
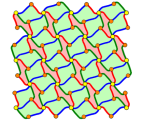
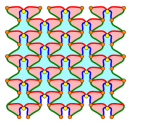
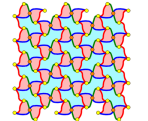
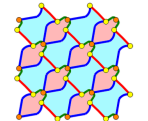
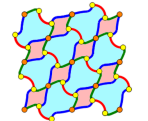
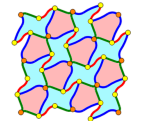
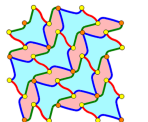
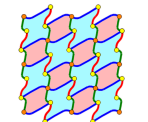
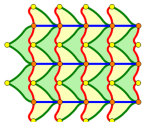
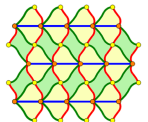
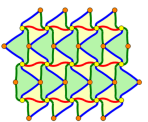
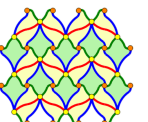
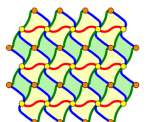
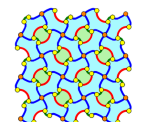
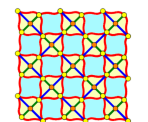
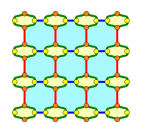
 31 $(2, 3, 1); 22^*$; $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle^{C_2}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{D_1} \rangle^{C_1}$	 32 $(2, 3, 1); * \times$; $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_1} \rangle^{C_1}$	 33 $(2, 3, 1); 2222$; $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle^{C_2}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle^{C_2}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_2}.4^{C_2} \rangle^{C_1}$	 34 $(2, 3, 1); 22 \times$; $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_2} \rangle^{C_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_2} \rangle^{C_1}$	 35 $(2, 3, 1); 2 * 22$; $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.8^{D_2} \rangle^{D_1}$ $\langle 3^{C_1}.3^{C_1}; 8^{D_2}.8^{D_2} \rangle^{C_2}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.8^{D_2} \rangle^{D_1}$
 36 $(2, 3, 1); 2 * 22$; $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.3^{D_1} \rangle^{C_2}$ $\langle 5^{D_1}.5^{D_1}; 4^{D_2}.4^{D_2} \rangle^{D_2}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.4^{D_2} \rangle^{D_1}$	 37 $(2, 3, 1); * \times$; $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.3^{C_1} \rangle^{C_1}$ $\langle 5^{C_1}.5^{C_1}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{D_1} \rangle^{C_1}$	 38 $(2, 3, 1); 22 \times$; $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ $\langle 5^{C_1}.5^{C_1}; 4^{C_2}.4^{C_2} \rangle^{C_2}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_2} \rangle^{C_1}$	 39 $(2, 3, 2); 22^*$; $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle^{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle^{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle^{C_1}$	 40 $(2, 3, 2); 442$; $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle^{C_2}$
 41 $(2, 3, 2); * \times$; $\langle 4^{C_1}.4^{D_1}; 4^{C_1}.4^{D_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{D_1}; 4^{C_1}.4^{D_1} \rangle^{C_1}$	 42 $(2, 3, 2); 22 \times$; $\langle 4^{C_1}.4^{C_2}; 4^{C_1}.4^{C_2} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_2}; 4^{C_1}.4^{C_2} \rangle^{C_1}$	 43 $(2, 3, 2); 22^*$; $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{C_2} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{C_2} \rangle^{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{C_2}.4^{C_2} \rangle^{C_1}$	 44 $(2, 3, 2); 22^*$; $\langle 4^{C_2}.4^{C_2}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 4^{C_2}.4^{C_2}; 4^{D_1}.4^{D_1} \rangle^{D_1}$ $\langle 4^{C_2}.4^{C_2}; 4^{D_1}.4^{D_1} \rangle^{C_1}$	 45 $(2, 3, 2); *2222$; $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle^{D_2}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle^{D_2}$ $\langle 6^{D_2}.6^{D_2}; 3^{D_1}.3^{D_1} \rangle^{C_1}$
 46 $(2, 3, 2); 3 * 3$; $\langle 6^{D_1}.6^{D_3}; 3^{C_1}.3^{C_1} \rangle^{D_1}$ $\langle 6^{D_1}.6^{D_3}; 3^{C_1}.3^{C_1} \rangle^{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_3} \rangle^{C_1}$	 47 $(2, 3, 2); *2222$; $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle^{D_2}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle^{D_2}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_2}.6^{D_2} \rangle^{C_1}$	 48 $(2, 3, 2); 3 * 3$; $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_3} \rangle^{D_1}$ $\langle 3^{C_1}.3^{C_3}; 6^{D_1}.6^{D_1} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_3} \rangle^{D_1}$	 49 $(2, 3, 2); * \times$; $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{D_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.6^{D_1} \rangle^{C_1}$ $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.6^{D_1} \rangle^{C_1}$	 50 $(2, 3, 2); 22 \times$; $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{C_2} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.6^{C_2} \rangle^{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.6^{C_2} \rangle^{C_1}$
 51 $(2, 3, 2); * \times$; $\langle 3^{C_1}.6^{D_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 3^{C_1}.6^{D_1}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 3^{C_1}.6^{D_1}; 4^{C_1}.4^{D_1} \rangle^{C_1}$	 52 $(2, 3, 2); 22 \times$; $\langle 3^{C_1}.6^{C_2}; 4^{C_1}.4^{C_1} \rangle^{C_1}$ $\langle 3^{C_1}.6^{C_2}; 4^{C_1}.4^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.6^{C_2}; 4^{C_1}.4^{C_2} \rangle^{C_1}$	 53 $(2, 3, 2); **$; $\langle 4^{D_1}.6^{D_1}; 3^{C_1}.4^{D_1} \rangle^{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle^{D_1}$ $\langle 4^{D_1}.6^{D_1}; 3^{C_1}.4^{D_1} \rangle^{C_1}$	 54 $(2, 3, 2); 22^*$; $\langle 4^{C_2}.6^{D_1}; 3^{C_1}.4^{C_2} \rangle^{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle^{D_1}$ $\langle 4^{C_2}.6^{D_1}; 3^{C_1}.4^{C_2} \rangle^{C_1}$	 55 $(2, 3, 2); 22^*$; $\langle 4^{D_1}.6^{C_2}; 3^{C_1}.4^{D_1} \rangle^{C_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ $\langle 4^{D_1}.6^{C_2}; 3^{C_1}.4^{D_1} \rangle^{C_1}$
 56 $(2, 3, 2); 22^*$; $\langle 4^{C_2}.6^{D_1}; 3^{C_1}.4^{D_1} \rangle^{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ $\langle 4^{C_2}.6^{D_1}; 3^{C_1}.4^{D_1} \rangle^{C_1}$	 57 $(2, 3, 2); 2222$; $\langle 4^{C_2}.6^{C_2}; 3^{C_1}.4^{C_2} \rangle^{C_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{C_1}.3^{C_1} \rangle^{C_2}$ $\langle 4^{C_2}.6^{C_2}; 3^{C_1}.4^{C_2} \rangle^{C_1}$	 58 $(2, 3, 2); **$; $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle^{D_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{D_1} \rangle^{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{D_1} \rangle^{C_1}$	 59 $(2, 3, 2); 22^*$; $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle^{D_1}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_2}.6^{D_1} \rangle^{C_1}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_2}.6^{D_1} \rangle^{C_1}$	 60 $(2, 3, 2); 22^*$; $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle^{C_2}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{C_2} \rangle^{C_1}$
 61 $(2, 3, 2); 22^*$; $\langle 3^{C_1}.4^{D_1}; 4^{C_2}.6^{D_1} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle^{C_2}$ $\langle 3^{C_1}.4^{D_1}; 4^{C_2}.6^{D_1} \rangle^{C_1}$	 62 $(2, 3, 2); 2222$; $\langle 3^{C_1}.4^{C_2}; 4^{C_2}.6^{C_2} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_2}.6^{C_2} \rangle^{C_2}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_2}.6^{C_2} \rangle^{C_1}$	 63 $(2, 3, 2); 4 * 2$; $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.4^{C_4} \rangle^{C_1}$ $\langle 4^{D_2}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle^{D_1}$ $\langle 4^{D_2}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle^{D_1}$	 64 $(2, 3, 2); 4 * 2$; $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.6^{D_1} \rangle^{D_1}$ $\langle 3^{C_1}.4^{C_2}; 6^{D_1}.6^{D_1} \rangle^{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.6^{D_1} \rangle^{D_1}$	 65 $(2, 3, 2); *222$; $\langle 8^{D_2}.8^{D_2}; 3^{D_1}.3^{D_1} \rangle^{D_2}$ $\langle 8^{D_2}.8^{D_2}; 3^{D_1}.3^{D_1} \rangle^{D_2}$ $\langle 4^{D_2}.8^{D_2}; 3^{D_1}.3^{D_1} \rangle^{C_1}$

TABLE 4. (continued)

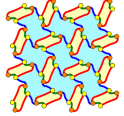
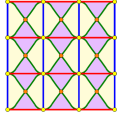
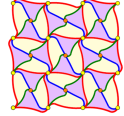
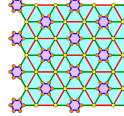
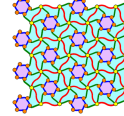
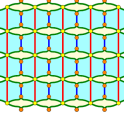
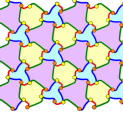
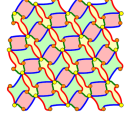
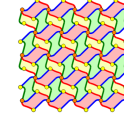
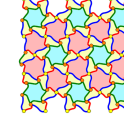
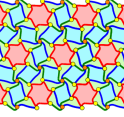
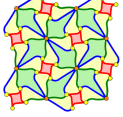
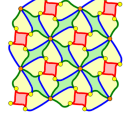
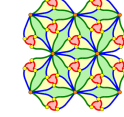
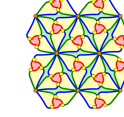
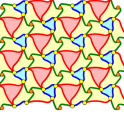
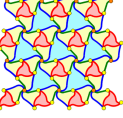
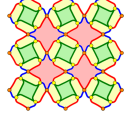
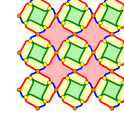
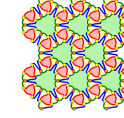
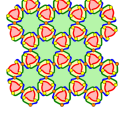
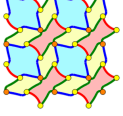
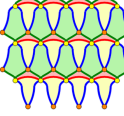
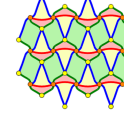
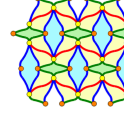
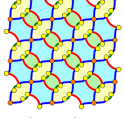
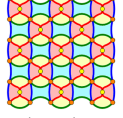
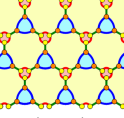
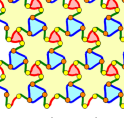
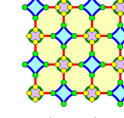
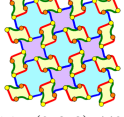
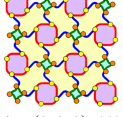
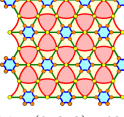
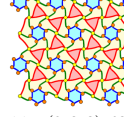
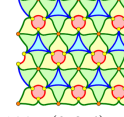
 66 $(2, 3, 2); 22\times;$ $\langle 8^{C_2}.8^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.8^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.8^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 67 $(2, 3, 2); *2222;$ $\langle 3^{D_1}.3^{D_1}; 8^{D_2}.8^{D_2} \rangle_{D_2}$ $\langle 3^{D_1}.3^{D_1}; 8^{D_2}.8^{D_2} \rangle_{D_2}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_2}.8^{D_2} \rangle_{C_1}$	 68 $(2, 3, 2); 22\times;$ $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.8^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 8^{C_2}.8^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.8^{C_2} \rangle_{C_1}$	 69 $(2, 3, 2); *632;$ $\langle 4^{D_1}.6^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 6^{D_3}.6^{D_3} \rangle_{D_2}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_3} \rangle_{D_1}$	 70 $(2, 3, 2); 632;$ $\langle 4^{C_1}.6^{C_6}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 6^{C_3}.6^{C_3} \rangle_{C_2}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{C_3} \rangle_{C_1}$
 71 $(2, 3, 2); *2222;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_2}$ $\langle 4^{D_1}.4^{D_1}; 6^{D_2}.6^{D_2} \rangle_{D_2}$ $\langle 4^{D_1}.4^{D_2}; 3^{D_1}.6^{D_2} \rangle_{C_1}$	 72 $(2, 3, 3); 333;$ $\langle 6^{C_3}.6^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_3}.6^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 6^{C_3}.6^{C_3}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 73 $(2, 3, 3); 22*;$ $\langle 4^{C_2}.4^{C_2}; 3^{C_1}.6^{D_1} \rangle_{C_1}$ $\langle 4^{C_2}.4^{D_1}; 3^{C_1}.6^{D_1} \rangle_{C_1}$ $\langle 4^{C_2}.4^{D_1}; 3^{C_1}.6^{D_1} \rangle_{C_1}$	 74 $(2, 3, 3); 2222;$ $\langle 4^{C_2}.4^{C_2}; 3^{C_1}.6^{C_2} \rangle_{C_1}$ $\langle 4^{C_2}.4^{C_2}; 3^{C_1}.6^{C_2} \rangle_{C_1}$ $\langle 4^{C_2}.4^{C_2}; 3^{C_1}.6^{C_2} \rangle_{C_1}$	 75 $(2, 3, 3); 632;$ $\langle 3^{C_1}.6^{C_6}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_3}; 4^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_3}; 4^{C_1}.4^{C_2} \rangle_{C_1}$
 76 $(2, 3, 3); 632;$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_6}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.6^{C_3} \rangle_{C_1}$	 77 $(2, 3, 3); 4 * 2;$ $\langle 3^{C_1}.4^{D_1}; 4^{C_1}.8^{D_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{C_1}.8^{D_2} \rangle_{C_1}$	 78 $(2, 3, 3); 442;$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.8^{C_4} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.8^{C_4} \rangle_{C_1}$	 79 $(2, 3, 3); 3 * 3;$ $\langle 3^{C_1}.4^{D_1}; 4^{C_1}.12^{D_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{C_1}.12^{D_3} \rangle_{C_1}$	 80 $(2, 3, 3); 632;$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.12^{C_6} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_2}; 4^{C_1}.12^{C_6} \rangle_{C_1}$
 81 $(2, 3, 3); 333;$ $\langle 3^{C_3}.4^{C_1}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 3^{C_3}.4^{C_1}; 5^{C_1}.5^{C_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.5^{C_1} \rangle_{C_1}$	 82 $(2, 3, 3); 333;$ $\langle 3^{C_1}.6^{C_3}; 4^{C_1}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_3}; 4^{C_1}.6^{C_3} \rangle_{C_1}$	 83 $(2, 3, 3); 4 * 2;$ $\langle 3^{C_1}.8^{D_2}; 4^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.8^{D_2}; 4^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	 84 $(2, 3, 3); 442;$ $\langle 3^{C_1}.8^{C_4}; 4^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.8^{C_4}; 4^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{C_4}; 4^{C_1}.4^{C_1} \rangle_{C_1}$	 85 $(2, 3, 3); 3 * 3;$ $\langle 3^{C_1}.12^{D_3}; 4^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_3}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_1}.12^{D_3}; 4^{C_1}.4^{D_1} \rangle_{C_1}$
 86 $(2, 3, 3); 632;$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.12^{C_6} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.12^{C_6} \rangle_{C_1}$	 87 $(2, 3, 3); 2 * 22;$ $\langle 4^{C_2}.6^{D_2}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 6^{D_2}.6^{D_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.6^{D_2}; 3^{C_1}.4^{D_1} \rangle_{C_1}$	 88 $(2, 3, 3); **;$ $\langle 3^{D_1}.4^{D_1}; 4^{D_1}.6^{D_1} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.4^{D_1}; 4^{D_1}.6^{D_1} \rangle_{C_1}$	 89 $(2, 3, 3); 22*;$ $\langle 3^{D_1}.4^{C_2}; 4^{D_1}.6^{C_2} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{C_2}.6^{C_2} \rangle_{D_1}$ $\langle 3^{D_1}.4^{C_2}; 4^{D_1}.6^{C_2} \rangle_{C_1}$	 90 $(2, 3, 3); 2 * 22;$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.6^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_1}.6^{D_1} \rangle_{C_2}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.6^{D_1} \rangle_{C_1}$
 91 $(2, 3, 3); *2222;$ $\langle 6^{D_2}.6^{D_2}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_2}.6^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 4^{D_2}.6^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	 92 $(2, 3, 3); 2 * 22;$ $\langle 3^{D_1}.4^{D_2}; 6^{C_2}.6^{C_2} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_2}.6^{C_2} \rangle_{D_1}$ $\langle 3^{D_1}.4^{D_2}; 6^{C_2}.6^{C_2} \rangle_{D_1}$	 93 $(2, 3, 3); *333;$ $\langle 3^{D_3}.12^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 3^{D_3}.12^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 12^{D_3}.12^{D_3}; 3^{D_1}.3^{D_1} \rangle_{D_1}$	 94 $(2, 3, 3); 333;$ $\langle 3^{C_3}.12^{C_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 3^{C_3}.12^{C_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 12^{C_3}.12^{C_3}; 3^{C_1}.3^{C_1} \rangle_{D_1}$	 95 $(2, 3, 3); *442;$ $\langle 4^{D_4}.8^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_3}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 4^{D_4}.8^{D_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$
 96 $(2, 3, 3); 442;$ $\langle 8^{C_4}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_2}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 97 $(2, 3, 3); 442;$ $\langle 8^{C_4}.8^{C_4}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_4}.8^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 4^{C_4}.8^{C_2}; 3^{C_1}.3^{C_1} \rangle_{C_1}$	 98 $(2, 3, 3); *632;$ $\langle 4^{D_1}.6^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 3^{D_3}.4^{D_1}; 6^{D_2}.6^{D_2} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_2} \rangle_{D_1}$	 99 $(2, 3, 3); 632;$ $\langle 4^{C_1}.6^{C_6}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 3^{C_3}.4^{C_1}; 6^{C_2}.6^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{C_2} \rangle_{C_1}$	 100 $(2, 3, 4); *333;$ $\langle 3^{D_1}.3^{D_3}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_3}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_3} \rangle_{C_1}$

TABLE 4. (continued)

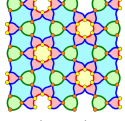
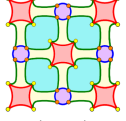
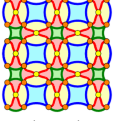
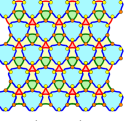
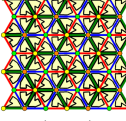
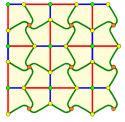
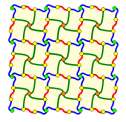
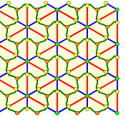
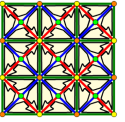
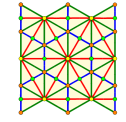
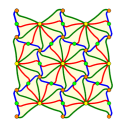
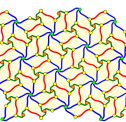
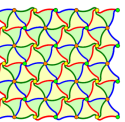
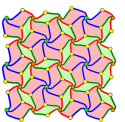
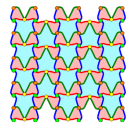
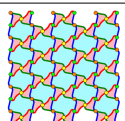
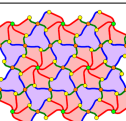
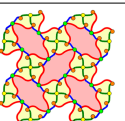
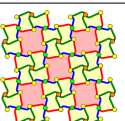
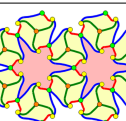
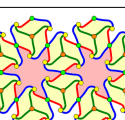
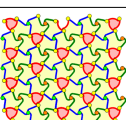
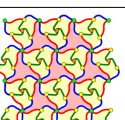
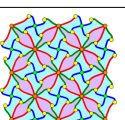
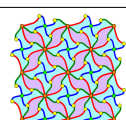
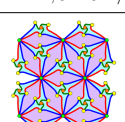
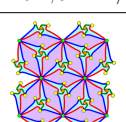
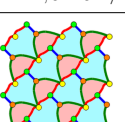
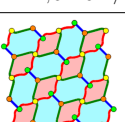
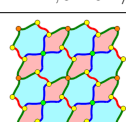
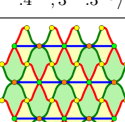
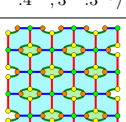
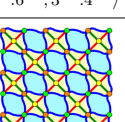
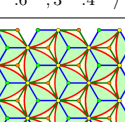
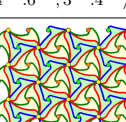
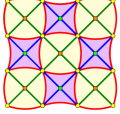
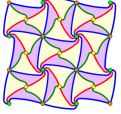
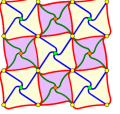
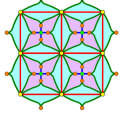
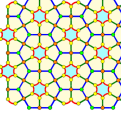
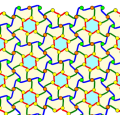
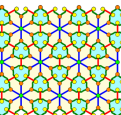
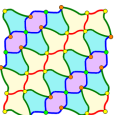
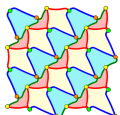
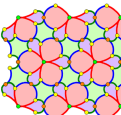
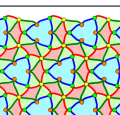
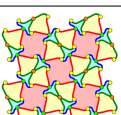
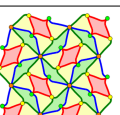
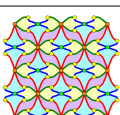
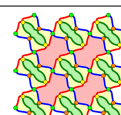
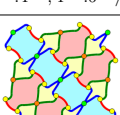
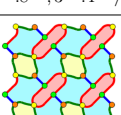
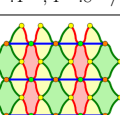
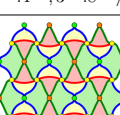
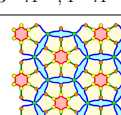
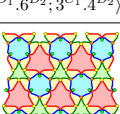
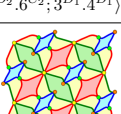
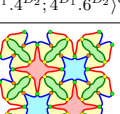
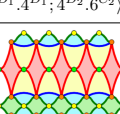
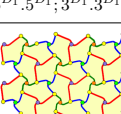
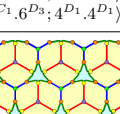
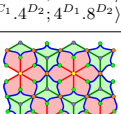
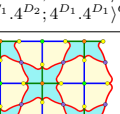
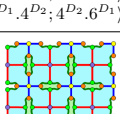
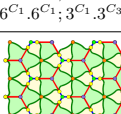
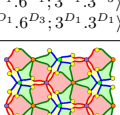
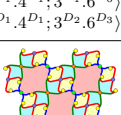
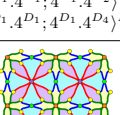
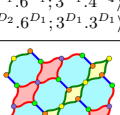
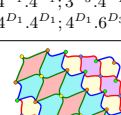
 101 (2, 3, 4); *632; $\langle 3^{D_1}.6^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{D_1}.6^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.6^{D_2}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 102 (2, 3, 4); *442; $\langle 4^{D_1}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_4}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 103 (2, 3, 4); *442; $\langle 3^{D_1}.4^{D_4}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_2}.6^{D_1} \rangle_{C_1}$ $\langle 3^{D_1}.4^{D_4}; 6^{D_1}.6^{D_1} \rangle_{D_1}$	 104 (2, 3, 4); *333; $\langle 4^{D_1}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{D_3}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 3^{D_3}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$	 105 (3, 3, 1); *333; $\langle 3^{C_1}.3^{C_1}; 6^{D_3}.6^{D_3} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_3}.6^{D_3} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_3}.6^{D_3} \rangle_{D_1}$
 106 (3, 3, 1); 4 * 2; $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_2} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_2} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.4^{D_2} \rangle_{C_1}$	 107 (3, 3, 1); 442; $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$	 108 (3, 3, 1); 3 * 3; $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.6^{D_3} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 4^{D_1}.6^{D_3} \rangle_{D_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_3}.4^{D_1} \rangle_{C_1}$	 109 (3, 3, 1); *442; $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.8^{D_4} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.8^{D_4} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 8^{D_4}.8^{D_4} \rangle_{D_1}$	 110 (3, 3, 1); *632; $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.6^{D_3} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.12^{D_6} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{D_3}.12^{D_6} \rangle_{D_1}$
 111 (3, 3, 1); 632; $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.12^{C_6} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_3}.12^{C_6} \rangle_{C_1}$	 112 (3, 3, 1); 632; $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.6^{C_6} \rangle_{C_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_2}.3^{C_1} \rangle_{C_2}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$	 113 (3, 3, 2); 333; $\langle 3^{C_1}.3^{C_1}; 6^{C_3}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_3}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 6^{C_3}.6^{C_3} \rangle_{C_1}$	 114 (3, 3, 2); 632; $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{C_6} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.6^{C_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.6^{C_3} \rangle_{C_1}$	 115 (3, 3, 2); 22*; $\langle 3^{C_1}.6^{D_1}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.6^{D_1}; 4^{C_2}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.6^{D_1}; 4^{C_2}.4^{D_1} \rangle_{C_1}$
 116 (3, 3, 2); 2222; $\langle 3^{C_1}.6^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.6^{C_2}; 4^{C_2}.4^{C_2} \rangle_{C_1}$	 117 (3, 3, 2); 632; $\langle 4^{C_1}.6^{C_3}; 3^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.6^{C_6} \rangle_{C_1}$ $\langle 4^{C_1}.6^{C_3}; 3^{C_1}.4^{C_2} \rangle_{C_1}$	 118 (3, 3, 2); 4 * 2; $\langle 4^{C_1}.8^{D_2}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_1}.8^{D_2}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$	 119 (3, 3, 2); 442; $\langle 4^{C_1}.8^{C_4}; 3^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.8^{C_4}; 3^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$	 120 (3, 3, 2); 3 * 3; $\langle 4^{C_1}.12^{D_3}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_1}.12^{D_3}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$
 121 (3, 3, 2); 632; $\langle 4^{C_1}.12^{C_6}; 3^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.12^{C_6}; 3^{C_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$	 122 (3, 3, 2); 333; $\langle 5^{C_1}.5^{C_1}; 3^{C_2}.4^{C_1} \rangle_{C_1}$ $\langle 3^{C_3}.5^{C_1}; 4^{C_1}.4^{C_1} \rangle_{C_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_3}.4^{C_1} \rangle_{C_1}$	 123 (3, 3, 2); 333; $\langle 4^{C_1}.6^{C_3}; 3^{C_1}.6^{C_3} \rangle_{C_1}$ $\langle 4^{C_1}.6^{C_3}; 3^{C_1}.6^{C_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$	 124 (3, 3, 2); 4 * 2; $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$ $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.8^{D_2} \rangle_{C_1}$ $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.8^{D_2} \rangle_{C_1}$	 125 (3, 3, 2); 442; $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.4^{C_4} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.8^{C_4} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.8^{C_4} \rangle_{C_1}$
 126 (3, 3, 2); 3 * 3; $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.12^{D_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{D_1}; 3^{C_1}.12^{D_3} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$	 127 (3, 3, 2); 632; $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.12^{C_6} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_2}; 3^{C_1}.12^{C_6} \rangle_{C_1}$ $\langle 4^{C_1}.4^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$	 128 (3, 3, 2); **; $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.6^{D_1}; 3^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.6^{D_1}; 3^{D_1}.4^{D_1} \rangle_{C_1}$	 129 (3, 3, 2); 22*; $\langle 6^{C_2}.6^{C_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.6^{C_2}; 3^{D_1}.4^{C_2} \rangle_{C_1}$ $\langle 4^{D_1}.6^{C_2}; 3^{D_1}.4^{C_2} \rangle_{C_1}$	 130 (3, 3, 2); 2 * 22; $\langle 4^{D_1}.6^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{C_1}.3^{C_1} \rangle_{C_2}$ $\langle 4^{D_1}.6^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$
 131 (3, 3, 2); 2 * 22; $\langle 3^{C_1}.3^{C_1}; 6^{D_2}.6^{D_2} \rangle_{D_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{C_2}.6^{D_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{C_2}.6^{D_2} \rangle_{C_1}$	 132 (3, 3, 2); 2 * 22; $\langle 6^{C_2}.6^{C_2}; 3^{D_1}.4^{D_2} \rangle_{D_1}$ $\langle 6^{C_2}.6^{C_2}; 3^{D_1}.4^{D_2} \rangle_{D_1}$ $\langle 4^{D_2}.6^{C_2}; 3^{D_1}.3^{D_1} \rangle_{C_1}$	 133 (3, 3, 2); *2222; $\langle 3^{C_1}.4^{D_2}; 6^{D_2}.6^{D_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.6^{D_2} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{D_2}.6^{D_2} \rangle_{D_1}$	 134 (3, 3, 2); *333; $\langle 3^{D_1}.3^{D_1}; 3^{D_3}.12^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 12^{D_3}.12^{D_3} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 3^{D_3}.12^{D_3} \rangle_{D_1}$	 135 (3, 3, 2); 333; $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.12^{C_3} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 12^{C_3}.12^{C_3} \rangle_{D_1}$ $\langle 3^{C_1}.3^{C_1}; 3^{C_3}.12^{C_3} \rangle_{D_1}$

TABLE 4. (continued)

 136 $(3, 3, 2); *442;$ $\langle 3^{D_1}.3^{D_1}; 4^{D_4}.8^{D_2} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 8^{D_2}.8^{D_2} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 4^{D_4}.8^{D_2} \rangle_{D_1}$	 137 $(3, 3, 2); 442;$ $\langle 3^{C_1}.3^{C_1}; 8^{C_4}.8^{C_4} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.8^{C_4} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_2}.8^{C_4} \rangle_{C_1}$	 138 $(3, 3, 2); 442;$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.8^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 8^{C_2}.8^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.3^{C_1}; 4^{C_4}.8^{C_2} \rangle_{C_1}$	 139 $(3, 3, 2); *442;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 3^{D_1}.3^{D_1}; 12^{D_4}.12^{D_4} \rangle_{D_2}$ $\langle 3^{D_1}.4^{D_1}; 3^{D_1}.12^{D_4} \rangle_{C_1}$	 140 $(3, 3, 2); *632;$ $\langle 5^{D_1}.5^{D_1}; 3^{D_3}.4^{D_2} \rangle_{D_1}$ $\langle 5^{D_1}.6^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.4^{D_2} \rangle_{D_1}$
 141 $(3, 3, 2); 632;$ $\langle 5^{C_1}.5^{C_1}; 3^{C_3}.4^{C_2} \rangle_{C_1}$ $\langle 5^{C_1}.6^{C_6}; 3^{C_1}.3^{C_1} \rangle_{C_1}$ $\langle 5^{C_1}.5^{C_1}; 3^{C_1}.4^{C_2} \rangle_{C_1}$	 142 $(3, 3, 2); *632;$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.6^{D_6} \rangle_{D_1}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_2}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.4^{D_1} \rangle_{C_1}$	 143 $(3, 3, 3); 2 * 22;$ $\langle 4^{D_1}.4^{D_2}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_2}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 144 $(3, 3, 3); * \times;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 145 $(3, 3, 3); 3 * 3;$ $\langle 4^{D_1}.6^{C_3}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.6^{D_3} \rangle_{C_1}$ $\langle 4^{D_1}.6^{C_3}; 3^{C_1}.4^{D_1} \rangle_{C_1}$
 146 $(3, 3, 3); 3 * 3;$ $\langle 3^{C_1}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{C_3} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.6^{C_3} \rangle_{C_1}$	 147 $(3, 3, 3); 4 * 2;$ $\langle 4^{D_1}.8^{C_4}; 3^{C_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.8^{C_4}; 3^{C_1}.4^{D_1} \rangle_{C_1}$	 148 $(3, 3, 3); 4 * 2;$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.8^{C_4} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_1}; 4^{D_1}.8^{C_4} \rangle_{C_1}$	 149 $(3, 3, 3); 2 * 22;$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{C_2}; 3^{C_1}.8^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{C_2}; 3^{C_1}.8^{D_2} \rangle_{C_1}$	 150 $(3, 3, 3); 2 * 22;$ $\langle 3^{C_1}.8^{D_2}; 4^{D_1}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.8^{D_2}; 4^{D_1}.4^{C_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$
 151 $(3, 3, 3); *2222;$ $\langle 4^{D_1}.6^{D_2}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 6^{D_2}.6^{D_2}; 3^{C_1}.3^{C_1} \rangle_{D_1}$ $\langle 4^{D_1}.6^{D_2}; 3^{C_1}.4^{D_2} \rangle_{C_1}$	 152 $(3, 3, 3); 2 * 22;$ $\langle 6^{C_2}.6^{C_2}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.6^{C_2}; 3^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_2}.6^{C_2}; 3^{D_1}.4^{D_1} \rangle_{C_1}$	 153 $(3, 3, 3); *2222;$ $\langle 3^{C_1}.3^{C_1}; 6^{D_2}.6^{D_2} \rangle_{D_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.6^{D_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.6^{D_2} \rangle_{C_1}$	 154 $(3, 3, 3); 2 * 22;$ $\langle 3^{D_1}.4^{D_1}; 4^{D_2}.6^{C_2} \rangle_{C_1}$ $\langle 3^{D_1}.3^{D_1}; 6^{C_2}.6^{C_2} \rangle_{D_1}$ $\langle 3^{D_1}.4^{D_1}; 4^{D_2}.6^{C_2} \rangle_{C_1}$	 155 $(3, 3, 3); 632;$ $\langle 4^{D_2}.5^{D_1}; 3^{D_1}.6^{D_3} \rangle_{C_1}$ $\langle 5^{D_1}.6^{D_6}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 5^{D_1}.5^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_1}$
 156 $(3, 3, 4); *333;$ $\langle 3^{C_1}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 6^{D_1}.6^{D_1}; 4^{D_1}.4^{D_1} \rangle_{D_1}$ $\langle 3^{C_1}.6^{D_3}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 157 $(3, 3, 4); *2222;$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.8^{D_2} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.8^{D_2} \rangle_{C_1}$	 158 $(3, 3, 4); *442;$ $\langle 3^{C_1}.8^{D_4}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.8^{D_4}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 3^{C_1}.4^{D_2}; 4^{D_1}.4^{D_1} \rangle_{C_1}$	 159 $(3, 3, 4); *2222;$ $\langle 3^{D_1}.3^{D_1}; 6^{D_1}.6^{D_1} \rangle_{D_1}$ $\langle 3^{D_1}.4^{D_2}; 4^{D_2}.6^{D_1} \rangle_{C_1}$ $\langle 3^{D_1}.4^{D_2}; 4^{D_2}.6^{D_1} \rangle_{C_1}$	 160 $(4, 3, 1); 333;$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_1} \rangle_{D_1}$ $\langle 6^{C_1}.6^{C_1}; 3^{C_1}.3^{C_3} \rangle_{C_1}$
 161 $(4, 3, 2); *333;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_3} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_3} \rangle_{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{C_1}$	 162 $(4, 3, 2); *632;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_3} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_1}.6^{D_3} \rangle_{D_1}$	 163 $(4, 3, 2); *442;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_4} \rangle_{D_1}$	 164 $(4, 3, 2); *442;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.4^{D_4} \rangle_{D_1}$ $\langle 4^{D_2}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{C_1}$	 165 $(4, 3, 2); *333;$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{D_3}.4^{D_1} \rangle_{D_1}$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.6^{D_3} \rangle_{C_1}$
 166 $(4, 3, 3); *333;$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.6^{D_3} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.6^{D_3} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.6^{D_3} \rangle_{C_1}$	 167 $(4, 3, 3); *2222;$ $\langle 4^{D_1}.8^{D_2}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.8^{D_2}; 3^{C_1}.4^{D_2} \rangle_{C_1}$	 168 $(4, 3, 3); *442;$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.8^{D_4} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_1}; 3^{C_1}.8^{D_4} \rangle_{C_1}$	 169 $(4, 3, 3); *2222;$ $\langle 6^{D_1}.6^{D_1}; 3^{D_1}.3^{D_1} \rangle_{D_1}$ $\langle 4^{D_2}.6^{D_1}; 3^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_2}.6^{D_1}; 3^{D_1}.4^{D_2} \rangle_{C_1}$	 170 $(4, 3, 4); *2222;$ $\langle 4^{D_1}.4^{D_1}; 4^{D_1}.4^{D_1} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_2}; 4^{D_1}.4^{D_2} \rangle_{C_1}$ $\langle 4^{D_1}.4^{D_2}; 4^{D_1}.4^{D_2} \rangle_{C_1}$

DEPARTMENT OF MATHEMATICS, ATENEO DE MANILA UNIVERSITY, QUEZON CITY,
PHILIPPINES

E-mail address: `mark.tomenes@obf.ateneo.edu`

DEPARTMENT OF MATHEMATICS, ATENEO DE MANILA UNIVERSITY, QUEZON CITY,
PHILIPPINES

E-mail address: `mdelaspenas@ateneo.edu`