Contributions to Discrete Mathematics

Volume 19, Number 3, Pages 154–162
ISSN 1715-0868

SOME RESULTS ABOUT STAR-FACTORS IN GRAPHS

SIZHONG ZHOU, YANG XU, AND ZHIREN SUN

ABSTRACT. For a set $\mathcal{S}$ of connected graphs, a subgraph F of a graph G is defined as an $\mathcal{S}$-factor of G if F satisfies that $V(F) = V(G)$ and every component of F is isomorphic to an element of $\mathcal{S}$. If every component of F is a star, then F is said to be a star-factor. A star-factor with size at most n may be written for a $\{K_{1,t} : 1 \leq t \leq n\}$-factor. A graph G is called a $\{K_{1,t} : 1 \leq t \leq n\}$-factor deleted graph if $G - e$ has a $\{K_{1,t} : 1 \leq t \leq n\}$-factor for every $e \in E(G)$. The sun toughness of a graph G is denoted by $s(G)$ and defined as follows:

$$s(G) = \min \left\{ \frac{|X|}{sun(G-X)} : X \subseteq V(G),\ sun(G-X) \geq 2 \right\}$$

if G is not a complete graph, and $s(G) = +\infty$ if G is a complete graph, where $sun(G - X)$ denotes the number of sun components of $G - X$. In this paper, we prove that (i) if G is a connected graph, and its sun toughness satisfies $s(G) \geq \frac{1}{n}$, then G admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor; (ii) if G is a $(k+1)$-connected graph, and its sun toughness $s(G) > \frac{k+1}{n+1}$, then $G - Y$ admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor for any $Y \subseteq V(G)$ with $|Y| = k$; (iii) if G is a 2-edge-connected graph, and its sun toughness $s(G) \geq \frac{1}{n-1}$, then G is a $\{K_{1,t} : 1 \leq t \leq n\}$-factor deleted graph. Furthermore, it is shown that our results are sharp.

1. INTRODUCTION

Graph theory plays an important role in chemical sciences. The basic layout of the graph theoretic model is a molecular structure in which vertices of the graph correspond to atoms, and edges correspond to chemical bonds. The study of this graph model supplies information on the chemical structure. One-factors or $\{K_{1,1}\}$-factors (also called special star-factors) in graphs correspond to Kekulé structures in chemistry. Kekulé structure possesses strong connections with organic chemistry, the enumeration of Kekulé structures in benzenoid molecules is a traditional and extensively elaborated field of mathematical chemistry [5]. Hosoya and Gutman [6] found a curious chemical relation between the Kekulé structure of hexagonal chains and the Hosoya index of a caterpillar tree. Xiao and Chen [16] found a curious chemical relation between the Kekulé structure of square-hexagonal chains and the Hosoya index of a caterpillar tree. Much other information on chemical structures can be extracted from the enumeration and classification of

Received by the editors May 09, 2021, and in revised form July 21, 2022.
2000 *Mathematics Subject Classification.* 05C70.
Key words and phrases. graph; star-factor; star-factor deleted graph; sun toughness.

Kekulé structures. The star-factor problem can be considered as a generalization of the well-known One-factor, $\{K_{1,1}\}$-factor or Kekulé structure problem, and possesses extensive applications in chemistry.

In this paper, we deal with only finite and undirected graphs, which have neither multiple edges nor loops. We denote by $G = (V(G), E(G))$ a graph, where $V(G)$ denotes the vertex set of G and $E(G)$ denotes the edge set of G. We denote the degree of a vertex x in G by $d_G(x)$. Let X be a subset of $V(G)$. We use $G - X$ to denote the subgraph obtained from G by deleting vertices in X together with the edges incident to vertices in X. Let E' be a subset of $E(G)$. We denote by $G - E'$ the subgraph obtained from G by deleting E'. Especially, we write $G - x = G - \{x\}$ for $x \in V(G)$ and $G - e = G - \{e\}$ for $e \in E(G)$. For a graph G, we use $\kappa(G)$ to denote its connectivity, and use $\lambda(G)$ to denote its edge-connectivity. Let m and n be two positive integers. A path of order n is denoted by P_n. We use K_n to denote a complete graph of order n and use $K_{m,n}$ to denote a complete bipartite graph with bipartite sets of order m and n. The graph $K_{1,n}$ is said to be a star. The vertex of degree n in $K_{1,n}$ is defined as the center when $n \geq 2$. For $K_{1,1}$, an arbitrary chosen vertex is its center.

For a set $\mathcal{S}$ of connected graphs, a subgraph F of a graph G is defined as an $\mathcal{S}$-factor of G if F satisfies that $V(F) = V(G)$ and if every component of F is isomorphic to an element of $\mathcal{S}$. If every component of F is a path, then F is said to be a path-factor. If every component of F is a star, then F is said to be a star-factor. Thus, the well-known perfect matching (i.e. 1-factor) is a P_2-factor or a $K_{1,1}$-factor. A star-factor with size at most n and at least m may be written for a $\{K_{1,t} : m \leq t \leq n\}$-factor. A graph G is called a $\{K_{1,t} : m \leq t \leq n\}$-factor deleted graph if $G - e$ has a $\{K_{1,t} : m \leq t \leq n\}$-factor for every $e \in E(G)$. In particular, a $\{K_{1,t} : m \leq t \leq n\}$-factor is a $\{K_{1,t} : 1 \leq t \leq n\}$-factor and a $\{K_{1,t} : m \leq t \leq n\}$-factor deleted graph is a $\{K_{1,t} : 1 \leq t \leq n\}$-factor deleted graph when $m = 1$.

Let $i(G)$ be the number of isolated vertices of G. Amahashi and Kano [1] and Las Vergnas [10] verified independently the following theorem on the existence of a $\{K_{1,t} : 1 \leq t \leq n\}$-factor, where $n \geq 2$ is an integer.

Theorem 1.1 (Amahashi and Kano [1], Las Vergnas [10])**.** *Let n be a positive integer with $n \geq 2$. Then a graph G admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor if and only if $i(G - X) \leq n|X|$ for any subset X of $V(G)$.*

Kano, Lu and Yu [7] justified the following theorem on the existence of a $\{K_{1,2}, K_{1,3}, K_5\}$-factor.

Theorem 1.2 (Kano, Lu and Yu [7])**.** *A graph G admits a $\{K_{1,2}, K_{1,3}, K_5\}$-factor if G satisfies $i(G - X) \leq \frac{1}{2}|X|$ for any subset X of $V(G)$.*

It is obvious that a graph with a $\{K_{1,2}, K_{1,3}, K_5\}$-factor includes a $\{K_{1,2}, K_{1,3}, K_{1,4}\}$-factor. Hence, a graph satisfying the condition of Theorem 1.2 admits a $\{K_{1,2}, K_{1,3}, K_{1,4}\}$-factor. Kano and Saito [8] presented a sufficient condition for a graph having a $\{K_{1,t} : m \leq t \leq 2m\}$-factor, which is an extension of the above observation.

Theorem 1.3 (Kano and Saito [8])**.** *Let G be a graph, and let $m \geq 2$ be an integer. Then G admits a $\{K_{1,t} : m \leq t \leq 2m\}$-factor if $i(G-X) \leq \frac{1}{m}|X|$ for any subset X of $V(G)$.*

It is easy to see that a $(\{K_{1,t} : m \leq t \leq 2m-1\} \cup \{K_{2m+1}\})$-factor is also a $\{K_{1,t} : m \leq t \leq 2m\}$-factor. Zhang, Yan and Kano [17] proved the following theorem on the existence of a $(\{K_{1,t} : m \leq t \leq 2m-1\} \cup \{K_{2m+1}\})$-factor, which is an improvement of Theorem 1.3.

Theorem 1.4 (Zhang, Yan and Kano [17])**.** *Let G be a graph, and let $m \geq 2$ be an integer. Then G admits a $(\{K_{1,t} : m \leq t \leq 2m-1\} \cup \{K_{2m+1}\})$-factor if G satisfies $i(G-X) \leq \frac{1}{m}|X|$ for any subset X of $V(G)$.*

A claw is a graph isomorphic to $K_{1,3}$. A graph is said to be a claw-free graph if it does not contain an induced claw. Kelmans [9] showed the following results on $\{K_{1,2}\}$-factors in claw-free graphs.

Theorem 1.5 (Kelmans [9])**.** *Let G be a 2-connected claw-free graph of order n. If $n \equiv 1$ (mod 3), then $G - x$ admits a $\{K_{1,2}\}$-factor (i.e. $\{P_3\}$-factor) for any $x \in V(G)$.*

Theorem 1.6 (Kelmans [9])**.** *Let G be a 2-connected claw-free graph of order n. If $n \equiv 0$ (mod 3), then $G - e$ admits a $\{K_{1,2}\}$-factor (i.e. $\{P_3\}$-factor) for any $e \in E(G)$.*

Note that $\{x\}$ is a vertex subset of G for any $x \in V(G)$, and $\{e\}$ is an edge subset of G for any $e \in E(G)$. Naturally, motivated by the above theorems, we consider the more general problems:

Problem 1. After deleting any k vertices of a graph G, does the remaining graph of G admit a star-factor? That is to say, does $G-U$ admit a star-factor for any $U \subseteq V(G)$ with $|U| = k$?

Problem 2. After deleting any m edges of a graph G, does the remaining graph of G admit a star-factor? That is to say, does $G - E'$ admit a star-factor for any $E' \subseteq E(G)$ with $|E'| = m$?

Our main results in this paper imply that the two problems above are true, which are shown in the following section. The other results on graph factors see [2, 3, 4, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

2. Main results and their proofs

A graph R is defined as a factor-critical graph if $R - x$ contains a perfect matching (i.e. $\{K_{1,1}\}$-factor) for every $x \in V(R)$. A graph H is defined as a sun if $H = K_1$, $H = K_2$ or H is the corona of a factor-critical graph R with order at least three, i.e., H is obtained from R by adding a new vertex $w = w(v)$ together with a new edge vw for every $v \in V(R)$. A sun of order

n with $n \geq 6$ is defined as a big sun. We use $sun(G)$ to denote the number of sun components of a graph G.

The sun toughness of a graph G is denoted by $s(G)$ and defined by

$$s(G) = \min\left\{\frac{|X|}{sun(G-X)} : X \subseteq V(G),\ sun(G-X) \geq 2\right\}$$

if G is not a complete graph; and $s(G) = +\infty$ if G is a complete graph. Then we study the relationship between sun toughness and star-factors with prescribed properties in graphs. Our main results in this paper are shown in the following.

Theorem 2.1. *Let G be a connected graph, and let n be an integer with $n \geq 2$. If its sun toughness $s(G) \geq \frac{1}{n}$, then G admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor.*

Proof. If G is a complete graph, it is obvious that G admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor. In the following, we assume that G is a non-complete graph. Suppose that G satisfies the hypothesis of Theorem 2.1, but it has no $\{K_{1,t} : 1 \leq t \leq n\}$-factor. In light of Theorem 1.1, there exists a vertex subset X of G satisfying

$$i(G-X) > n|X|.$$

Since G is a connected graph, $X \neq \emptyset$. Hence, $i(G-X) > n|X| \geq n \geq 2$. Combining this with $sun(G-X) \geq i(G-X)$ and the definition of $s(G)$, we get

$$s(G) \leq \frac{|X|}{sun(G-X)} \leq \frac{|X|}{i(G-X)} < \frac{|X|}{n|X|} = \frac{1}{n},$$

which contradicts that $s(G) \geq \frac{1}{n}$. Theorem 2.1 is verified. □

Theorem 2.2. *Let k and n be two integers with $k \geq 1$ and $n \geq 2$, and let G be a $(k+1)$-connected graph. If its sun toughness $s(G) > \frac{k+1}{n+1}$, then $G - Y$ admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor for any $Y \subseteq V(G)$ with $|Y| = k$.*

Proof. We write $G' = G - Y$. Assume that G' has no $\{K_{1,t} : 1 \leq t \leq n\}$-factor. Then it follows from Theorem 1.1 that

$$i(G'-X) \geq n|X| + 1 \tag{1}$$

for some vertex subset X of G'.

Claim 1. $X \neq \emptyset$.

Let $X = \emptyset$. Note that G is a $(k+1)$-connected graph and $|Y| = k$. Thus, $G' = G - Y$ is connected. Hence, $i(G') = 0$. Combining this with (1) and $X = \emptyset$, we get

$$0 = i(G') = i(G'-X) \geq n|X| + 1 = 1,$$

which is a contradiction. This completes the proof of Claim 1.

In the following, we consider two cases by the value of $sun(G'-X)$.

Case 1. $sun(G'-X) \leq 1$.

Note that $i(G'-X) \leq sun(G'-X) \leq 1$. In terms of (1) and Claim 1, we obtain

$$1 \geq sun(G'-X) \geq i(G'-X) \geq n|X|+1 \geq n+1 > 1,$$

a contradiction.

Case 2. $sun(G'-X) \geq 2$.

Note that $sun(G-(X\cup Y)) = sun(G'-X)$. According to the hypothesis of Theorem 2.2, the definition of $s(G)$, $sun(G'-X) \geq i(G'-X)$ and (1), we obtain

$$\begin{aligned}\frac{k+1}{n+1} &< s(G)\\ &\leq \frac{|X\cup Y|}{sun(G-(X\cup Y))}\\ &= \frac{|X|+k}{sun(G'-X)}\\ &\leq \frac{|X|+k}{i(G'-X)}\\ &\leq \frac{|X|+k}{n|X|+1},\end{aligned}$$

which implies

$$|X| < 1,$$

which contradicts Claim 1. This completes the proof of Theorem 2.2. □

Theorem 2.3. *Let n be a positive integer with $n \geq 2$, and let G be a 2-edge-connected graph. Then G is a $\{K_{1,t} : 1 \leq t \leq n\}$-factor deleted graph if its sun toughness $s(G) \geq \frac{1}{n-1}$.*

Proof. Obviously, Theorem 2.3 holds for a complete graph. Next, we may assume that G is a non-complete graph.

Let $G' = G - e$ for any $e \in E(G)$. In order to prove Theorem 2.3, we only need to verify that G' admits a $\{K_{1,t} : 1 \leq t \leq n\}$-factor. By contradiction, suppose that G' has no $\{K_{1,t} : 1 \leq t \leq n\}$-factor. Then it follows from Theorem 1.1 that

$$i(G'-X) \geq n|X|+1 \tag{2}$$

for some $X \subseteq V(G')$. In the following, we consider two cases for the value of $|X|$.

Case 1. $|X| = 0$.

In terms of (2), we obtain

$$i(G') \geq 1. \tag{3}$$

On the other hand, since G a 2-edge-connected graph, $G' = G - e$ is an edge-connected graph, which implies

$$i(G') = 0,$$

contradicting (3).

Case 2. $|X| \geq 1$.

Note that $i(G'-X) \leq sun(G'-X) = sun(G-X-e) \leq sun(G-X)+2$, which implies

$$sun(G-X) \geq i(G'-X) - 2. \tag{4}$$

Subcase 2.1. $sun(G-X) \geq 2$.

In light of (2), (4), the definition of $s(G)$ and the hypothesis of Theorem 2.3, we obtain

$$\begin{aligned} \frac{1}{n-1} &\leq s(G) \\ &\leq \frac{|X|}{sun(G-X)} \\ &\leq \frac{|X|}{i(G'-X)-2} \\ &\leq \frac{|X|}{n|X|+1-2} \\ &= \frac{|X|}{n|X|-1}, \end{aligned}$$

and thus

$$\frac{1}{n-1} \leq \frac{|X|}{n|X|-1}. \tag{5}$$

Let $f(|X|) = \frac{|X|}{n|X|-1}$. Then we have

$$f'(|X|) = \Big(\frac{|X|}{n|X|-1}\Big)' = -\frac{1}{(n|X|-1)^2} < 0,$$

which implies that $f(|X|)$ attains its maximum value at $|X| = 1$. Combining this with (5), we get

$$\frac{1}{n-1} \leq s(G) \leq \frac{|X|}{sun(G-X)} \leq \frac{|X|}{i(G'-X)-2} \leq \frac{1}{n-1},$$

which implies

$$sun(G-X) = i(G'-X) - 2. \tag{6}$$

Note that $G' = G - e$. Hence, $i(G'-X) \leq i(G-X)+2$. Combining this with (6) and $sun(G-X) \geq i(G-X)$, we obtain

$$i(G-X) \leq sun(G-X) = i(G'-X) - 2 \leq i(G-X) + 2 - 2 = i(G-X),$$

which implies

$$i(G-X) = i(G'-X) - 2. \tag{7}$$

It follows from (7) and $G' = G - e$ that $G - X$ has at least a component K_2. Thus, we have

$$sun(G-X) \geq i(G-X) + 1 = i(G'-X) - 2 + 1 = i(G'-X) - 1,$$

which contradicts (6).

Subcase 2.2. $sun(G-X) \leq 1$.

According to (2), (4) and $n \geq 2$, we have

$$sun(G-X) \geq i(G'-X) - 2 \geq n|X| + 1 - 2 \geq n - 1 \geq 1.$$

Combining this with $sun(G-X) \leq 1$, we get

$$sun(G-X) = 1 \tag{8}$$

and

$$i(G'-X) = 3. \tag{9}$$

We write $e = xy$. Let v_1, v_2, v_3 be three isolated vertices in $G' - X$. If $x, y \in V(G) \setminus \{v_1, v_2, v_3\}$, then by (9) we have

$$sun(G-X) \geq 3,$$

which contradicts (8).

If $x \in \{v_1, v_2, v_3\}$ and $y \in V(G) \setminus \{v_1, v_2, v_3\}$ (or $x \in V(G) \setminus \{v_1, v_2, v_3\}$ and $y \in \{v_1, v_2, v_3\}$), then it follows from (8) and (9) that

$$1 = sun(G-X) \geq 2,$$

which is a contradiction.

If $x, y \in \{v_1, v_2, v_3\}$, then from (9), it is easy to see that there are at least two sun components K_1 and K_2 in $G - X$, that is,

$$sun(G-X) \geq 2,$$

which contradicts (8). Theorem 2.3 is proved. $\square$

3. Remarks

Remark 1. The condition $s(G) \geq \frac{1}{n}$ in Theorem 2.1 is sharp. In order to prove this, we construct a graph $G = K_r \vee (rn+1)K_1$, where $\vee$ means "join", and r, n are two positive integers with $n \geq 2$. Obviously, $s(G) = \frac{r}{rn+1} < \frac{r}{rn} = \frac{1}{n}$. But $S(G) \to \frac{1}{n}$ when $r \to +\infty$. Let $X = V(K_r)$. Then $|X| = r$ and

$$i(G-X) = rn + 1 = n|X| + 1 > n|X|.$$

In light of Theorem 1.1, G has no $\{K_{1,t} : 1 \leq t \leq n\}$-factor.

Remark 2. We show that $s(G) > \frac{k+1}{n+1}$ in Theorem 2.2 is best possible. We construct a graph $G = K_{k+1} \vee (n+1)K_1$, where k, n are two positive integers with $n \geq 2$ and $\vee$ means "join". It is obvious that $s(G) = \frac{k+1}{n+1}$ and G is a $(k+1)$-connected graph. Set $Y = V(K_k) \subseteq V(K_{k+1})$ and $G' = G - Y$. Let $X = V(K_{k+1}) \setminus Y = V(K_1)$. Then $|X| = 1$ and

$$i(G'-X) = n + 1 = n|X| + 1 > n|X|.$$

In view of Theorem 1.1, $G' = G - Y$ has no $\{K_{1,t} : 1 \leq t \leq n\}$-factor.

Remark 3. In the following, we show that the hypothesis $s(G) \geq \frac{1}{n-1}$ in Theorem 2.3 cannot be replaced by $s(G) \geq \frac{1}{n}$.

Let $m \geq 2$ be an integer. We construct a graph $G = K_m \vee ((mn-1)K_1 \cup K_2)$, where $\vee$ means "join". It is easy to see that $s(G) = \frac{m}{(mn-1)+1} = \frac{1}{n}$ and

G is an m-edge-connected graph. Let $G' = G - e$, where $e \in E(K_2)$. We choose $X = V(K_m)$, and so $|X| = m$. Then we have

$$i(G' - X) = (mn - 1) + 2 = mn + 1 = n|X| + 1 > n|X|.$$

In terms of Theorem 1.1, $G' = G - e$ has no $\{K_{1,t} : 1 \le t \le n\}$-factor, that is, G is not a $\{K_{1,t} : 1 \le t \le n\}$-factor deleted graph.

Acknowledgements

The authors would like to thank the anonymous referees for their comments and suggestions on this paper.

References

1. A. Amahashi, M. Kano, On factors with given components, Discrete Mathematics 42(1982)1–6.
2. W. Gao, J. Guirao, Y. Chen, A toughness condition for fractional (k, m)-deleted graphs revisited, Acta Mathematica Sinica, English Series 35(2019)1227–1237.
3. W. Gao, W. Wang, D. Dimitrov, Toughness condition for a graph to be all fractional (g, f, n)-critical deleted, Filomat 33(9)(2019)2735–2746.
4. G. Gunther, B. Hartnell, D. Rall, Star-Factors and k-Bounded Total Domination, networks 27(1996)197–201.
5. I. Gutman, S. Cyvin, Introduction to the Theory of Benzenoid Hydrocarbons, Springer, Berlin, 1989.
6. H. Hosoya, I. Gutman, Kekulé structures of hexagonal chains-some unusual connections, Journal of Mathematical Chemistry 44(2008)559–568.
7. M. Kano, H. Lu, Q. Yu, Component factors with large components in graphs, Applied Mathematics Letters 23(2010)385–389.
8. M. Kano, A. Saito, Star-factors with large components, Discrete Mathematics 312(2012)2005–2008.
9. A. Kelmans, Packing 3-vertex paths in claw-free graphs and related topics, Discrete Applied Mathematics 159(2011)112–127.
10. M. Las Vergnas, An extension of Tutte's 1-factor theorem, Discrete Mathematics 23(1978)241–255.
11. X. Lv, A degree condition for graphs being fractional (a, b, k)-critical covered, Filomat 37(10)(2023)3315–3320.
12. S. Wang, W. Zhang, Isolated toughness for path factors in networks, RAIRO-Operations Research 56(4)(2022)2613–2619.
13. S. Wang, W. Zhang, On k-orthogonal factorizations in networks, RAIRO-Operations Research 55(2)(2021)969–977.
14. S. Wang, W. Zhang, Research on fractional critical covered graphs, Problems of Information Transmission 56(3)(2020)270–277.
15. J. Wu, Path-factor critical covered graphs and path-factor uniform graphs, RAIRO-Operations Research 56(6)(2022)4317–4325.
16. C. Xiao, H. Chen, Kekulé structures of suqare-hexagonal chains and the Hosoya index of caterpillar trees, Discrete Mathematics 339(2016)506–510.
17. Y. Zhang, G. Yan, M. Kano, Star-like factors with large components, Journal of the Operations Research Society of China 3(2015)81–88.
18. S. Zhou, A neighborhood union condition for fractional (a, b, k)-critical covered graphs, Discrete Applied Mathematics 323(2022)343–348.
19. S. Zhou, Q. Pan, L. Xu, Isolated toughness for fractional (2,b,k)-critical covered graphs, Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science 24(1)(2023)11-18.

20. S. Zhou, A result on fractional (a, b, k)-critical covered graphs, Acta Mathematicae Applicatae Sinica-English Series 37(4)(2021)657–664.
21. S. Zhou, Degree conditions and path factors with inclusion or exclusion properties, Bulletin Mathematique de la Societe des Sciences Mathematiques de Roumanie 66(1)(2023)3–14.
22. S. Zhou, Path factors and neighborhoods of independent sets in graphs, Acta Mathematicae Applicatae Sinica-English Series, 39(2)(2023)232-238.
23. S. Zhou, Remarks on restricted fractional (g, f)-factors in graphs, Discrete Applied Mathematics, 354(2024)271-278.
24. S. Zhou, Q. Bian, Q. Pan, Path factors in subgraphs, Discrete Applied Mathematics 319(2022)183–191.
25. S. Zhou, H. Liu, Discussions on orthogonal factorizations in digraphs, Acta Mathematicae Applicatae Sinica-English Series 38(2)(2022)417–425.
26. S. Zhou, H. Liu, Two sufficient conditions for odd $[1, b]$-factors in graphs, Linear Algebra and its Applications 661(2023)149–162.
27. S. Zhou, H. Liu, Y. Xu, A note on fractional ID-$[a, b]$-factor-critical covered graphs, Discrete Applied Mathematics 319(2022)511–516.
28. S. Zhou, Z. Sun, H. Liu, On $P_{\geq 3}$-factor deleted graphs, Acta Mathematicae Applicatae Sinica-English Series 38(1)(2022)178–186.
29. S. Zhou, Z. Sun, H. Liu, Some sufficient conditions for path-factor uniform graphs, Aequationes Mathematicae, 97(3)(2023)489-500.
30. S. Zhou, J. Wu, Q. Bian, On path-factor critical deleted (or covered) graphs, Aequationes Mathematicae 96(4)(2022)795–802.
31. S. Zhou, J. Wu, Y. Xu, Toughness, isolated toughness and path factors in graphs, Bulletin of the Australian Mathematical Society 106(2)(2022)195–202.

School of Science, Jiangsu University of Science and Technology,
Zhenjiang, Jiangsu 212100, China
E-mail address: `zsz_cumt@163.com`

Department of Mathematics, Qingdao Agricultural University, Qingdao,
Shandong 266109, China
E-mail address: `xuyang_825@126.com`

School of Mathematical Sciences, Nanjing Normal University, Nanjing,
Jiangsu 210023, China
E-mail address: `05119@njnu.edu.cn`