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CORRIGENDUM TO "ON THE ENUMERATION OF A CLASS OF TOROIDAL GRAPHS" [CONTRIB. DISCRETE MATH. 13 (2018), NO. 1, 79–119]

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Here, all the notations and definitions are given in [1]. Lemma [1, Lemma 5.6, p. 99] is not true in general on higher number of vertices for some cases. For example, $T(10,4,2)$ and $T(10,4,4)$ are not isomorphic although $(a_{1,1}, a_{1,2}) = (a_{2,t_1}, a_{2,t_2}) = (10, 20)$ since the cycle lengths of type B_2 (defined below) are different. Lemma [1, Lemma 5.6] can be modified in a similar way to [1, Lemma 4.10]. This skipped our attention while writing the paper.

Let M_1 and M_2 be two maps of type $\{3^2, 4, 3, 4\}$ with the same number of vertices on the torus and $T_i = T(r_i, s_i, k_i), i \in \{1, 2\}$ denote M_i. Let $C_{i,1}$ and $C_{i,2}$ denote nonhomologous cycles of type B_1 in $T(r_i, s_i, k_i)$. Define a cycle of type, say B_2 (as defined in [1, eqn. (4.1), p. 90]) by two paths which are parts of an upper horizontal cycle and a vertical cycle which is nonhomologous to horizontal cycles of type B_1. Let $C_{(i,k),2}$ denote a cycle of type B_2 in $T(r_i, s_i, k_i)$ if $C_{i,k}$ is horizontal in $T(r_i, s_i, k_i)$. More precisely, the cycle $C_{(i,k),2}$ is defined by $C_{i,k}$ and C_{i,k^1}, where $C_{i,k}$ is horizontal cycle and C_{i,k^1} is the vertical cycle for $k, k^1 \in \{1, 2\}$ and $k \neq k^1$ in T_i. A similar definition is also given in the proof of [1, Lemma 5.6] on page 99 for fixed horizontal and vertical cycles. Let $a_{i,j} = \text{length}(C_{i,j})$ and $a_{(i,k),2} = \text{length}(C_{(i,k),2})$. We use these in the following lemma which replaces [1, Lemma 5.6].

Lemma 5.6$'$. *The map $M_1 \cong M_2$ if and only if $\{a_{1,1},\ a_{1,2}\} = \{a_{2,1},\ a_{2,2}\}$ and $a_{(1,1),2} = a_{(2,t),2}$, where $a_{1,1} = a_{2,t}$ and $t \in \{1, 2\}$.*

The proof of Lemma 5.6$'$ is similar to that of [1, Lemma 5.6]. In Lemma 5.6$'$ we added lengths of the cycles of type B_2 which are not used in the statement of [1, Lemma 5.6]. Therefore, in [1, Cases 2, 3, and 4, Proof of Lemma 5.6], assume that $a_{(1,1),2} = a_{(2,t),2}$ where $a_{1,1} = a_{2,t}$ and $t \in \{1, 2\}$ which is equivalent condition, "length($C_3(1)$) = length($C_3(2)$)" in the cases.

Similar modifications are also needed in the lemmas [1, Lemmas 6.3, 7.3, 8.3, 9.3, 10.3, 11.4]. Here are the modified statements.

Lemma 6.3$'$. *The map $M_1 \cong M_2$ if and only if $\{a_{1,1},\ a_{1,2},\ a_{1,3}\} = \{a_{2,1},\ a_{2,2},\ a_{2,3}\}$ and $a_{(1,1),2} = a_{(2,t),2}$, where $a_{1,1} = a_{2,t}$ and $t \in \{1, 2, 3\}$.*

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Lemma 7.3$'$. *The map* $M_1 \cong M_2$ *if and only if* $\{b_{1,1},\, b_{1,2},\, b_{1,3}\} = \{b_{2,1},\, b_{2,2},\, b_{2,3}\}$ *and* $b_{(1,1),2} = b_{(2,t),2}$ *where* $b_{1,1} = b_{2,t}$ *and* $t \in \{1,2,3\}$.

Lemma 8.3$'$. *The map* $M_1 \cong M_2$ *if and only if* $\{a_{1,1},\, a_{1,2},\, a_{1,3}\} = \{a_{2,1},\, a_{2,2},\, a_{2,3}\}$ *and* $a_{(1,1),2} = a_{(2,t),2}$ *where* $a_{1,1} = a_{2,t}$ *and* $t \in \{1,2,3\}$.

Lemma 9.3$'$. *The map* $M_1 \cong M_2$ *if and only if* $\{b_{1,1},\, b_{1,2},\, b_{1,3}\} = \{b_{2,1},\, b_{2,2},\, b_{2,3}\}$ *and* $b_{(1,1),2} = b_{(2,t),2}$ *where* $b_{1,1} = b_{2,t}$ *and* $t \in \{1,2,3\}$.

Lemma 10.3$'$. *The map* $M_1 \cong M_2$ *if and only if* $\{c_{1,1},\, c_{1,2},\, c_{1,3}\} = \{c_{2,1},\, c_{2,2},\, c_{2,3}\}$ *and* $c_{(1,1),2} = c_{(2,t),2}$ *where* $c_{1,1} = c_{2,t}$ *and* $t \in \{1,2,3\}$.

Lemma 11.4$'$. *The map* $M_1 \cong M_2$ *if and only if* $\{a_{1,1},\, a_{1,2}\} = \{a_{2,1},\, a_{2,2}\}$ *and* $a_{(1,1),2} = a_{(2,t),2}$ *where* $a_{1,1} = a_{2,t}$ *and* $t \in \{1,2\}$.

The proofs of these lemmas would be same as mentioned above for Lemma 5.6$'$. Accordingly, similar changes would appear in [1, Tables 2-8].

In [1, Lemmas 6.2, 7.2, 10.2 11.3], we need to change the following ambiguities.

(1) In [1, Lemma 6.2 (v)] "$k \in \{2t+6 : 0 \le t \le \frac{r-10}{2}\} \setminus \{2(\frac{r-10}{4})+6\}$ if $s=1$" needs to be replaced by "$k \in \{2t+6 : 0 \le t \le \frac{r-10}{2}\} \setminus \{2(\frac{r-10}{4})+6 : 4 \mid (r-10)\}$ if $s=1$".

(2) In [1, Lemma 7.2 (v)] "$k \in \{4t+9 : 0 \le t \le \frac{r-20}{4}\} \setminus \{4(\frac{r}{8}-3)+9\}$ if $s=1$" needs to be replaced by "$k \in \{4t+9 : 0 \le t \le \frac{r-20}{4}\} \setminus \{4(\frac{r}{8}-3)+9 : 8 \mid r\}$ if $s=1$".

(3) In [1, Lemma 10.2] "(vi) $k \in \{3t+4 : 0 \le t \le \frac{r-9}{3}\}$ if $s=2$ & $k \in \{3t+1 : 0 \le t \le \frac{r-3}{3}\}$ if $s \ge 4$" needs to be replaced by "(v) $k \in \{3t+3 : 0 \le t \le \frac{r-9}{3}\}$ if $s=2$ & $k \in \{3t : 0 \le t \le \frac{r-3}{3}\}$ if $s \ge 4$".

(4) In [1, Lemma 11.3 (v)] "$k \in \{4t+6 : 0 \le t \le \frac{r-12}{4}\}$ if $s=1$" needs to be replaced by "$k \in \{4t+6 : 0 \le t \le \frac{r-12}{4}\} \setminus \{4 \times \frac{r-12}{8}+6 : 8 \mid (r-12)\}$ if $s=1$".

(5) In [1, Lemma 11.3 (v)] "$k \in \{4t-1 \pmod r : 0 \le t \le \frac{r-4}{4}\}$ if $s \ge 3$' needs to be replaced by "$k \in \{4t-1 \pmod r : 0 \le t \le \frac{r-4}{4}\}$ if $s \ge 3$, even and $k \in \{4t-2 \pmod r : 0 \le t \le \frac{r-4}{4}\}$ if $s \ge 3$, odd".

References

1. D. Maity and A. K. Upadhyay, *On the enumeration of a class of toroidal graphs*, Contrib. Discrete Math. **13** (2018), no. 1, 79–119.

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