Contributions to Discrete Mathematics

Volume 15, Number 1, Pages 22–41
ISSN 1715-0868

CONSTRUCTION OF STRONGLY REGULAR GRAPHS HAVING AN AUTOMORPHISM GROUP OF COMPOSITE ORDER

DEAN CRNKOVIĆ AND MARIJA MAKSIMOVIĆ

ABSTRACT. In this paper we outline a method for constructing strongly regular graphs from orbit matrices admitting an automorphism group of composite order. In 2011, C. Lam and M. Behbahani introduced the concept of orbit matrices of strongly regular graphs and developed an algorithm for the construction of orbit matrices of strongly regular graphs with a presumed automorphism group of prime order, and construction of corresponding strongly regular graphs. The method of constructing strongly regular graphs developed and employed in this paper is a generalization of that developed by C. Lam and M. Behbahani. Using this method we classify SRGs with parameters (49,18,7,6) having an automorphism group of order six. Eleven of the SRGs with parameters (49,18,7,6) constructed in that way are new. We obtain an additional 385 new SRGs(49,18,7,6) by switching. Comparing the constructed graphs with previously known SRGs with these parameters, we conclude that up to isomorphism there are at least 727 SRGs with parameters (49,18,7,6). Further, we show that there are no SRGs with parameters (99,14,1,2) having an automorphism group of order six or nine, i.e. we rule out automorphism groups isomorphic to Z_6, S_3, Z_9, or E_9.

1. Introduction

One of the main problems in the theory of strongly regular graphs (SRGs) is constructing and classifying SRGs with given parameters. A frequently used method of constructing combinatorial structures is a construction of combinatorial structures with a prescribed automorphism group. Orbit matrices of block designs have been used for such a construction of combinatorial designs since 1980s. However, orbit matrices of strongly regular graphs have not been introduced until 2011. Namely, Majid Behbahani and Clement Lam introduced the concept of orbit matrices of strongly regular

Received by the editors February 18, 2018 and in revised form March 2, 2019.
2010 *Mathematics Subject Classification.* 05E30, 20D45.
Key words and phrases. Strongly regular graph, automorphism group, orbit matrix.
This work has been fully supported by Croatian Science Foundation under the project 6732.

graphs in [2]. They developed an algorithm for construction of orbit matrices of strongly regular graphs with an automorphism group of prime order, and construction of corresponding strongly regular graphs. In this way they constructed strongly regular graphs with parameters (49,18,7,6) having an automorphism of order five or seven.

In this paper we present a method of constructing strongly regular graphs admitting an automorphism group of composite order from orbit matrices. This method uses the notion of an orbit matrix of a strongly regular graph introduced by Behbahani and Lam in [2]. Further, we give a classification of strongly regular graphs with parameters (49,18,7,6) having an automorphism group of order six. In that way we have constructed 55 SRGs with parameters (49,18,7,6), and eleven of them are new. Using Godsil–McKay switching and cycle-type switching we obtain additional 385 new SRGs with parameters (49,18,7,6), thereby proving that up to isomorphism there are at least 727 strongly regular graphs with parameters (49,18,7,6). Additionally, we use the described method of the construction of SRGs admitting an automorphism group of composite order for an attempt to construct a strongly regular graph with parameters (99,14,1,2) having an automorphism group of order six or nine, which would prove the existence of a strongly regular graph with these parameters.

The paper is organized as follows: after a brief description of the terminology and some background results, in Section 3 we describe the concept of orbit matrices, based on the work of Behbahani and Lam [2]. In Section 4 we explain the construction of strongly regular graphs from their orbit matrices. In Section 5 we apply this method to construct strongly regular graphs with parameters (49,18,7,6) having an automorphism group isomorphic to Z_6 or S_3, and in Section 6 we construct new SRGs with parameters (49,18,7,6) by switching. In Section 7 we show that there are no strongly regular graph with parameters (99,14,1,2) having an automorphism group isomorphic to Z_6, S_3, Z_9, or E_9.

2. Background and terminology

We assume that the reader is familiar with basic notions from the theory of finite groups. For basic definitions and properties of strongly regular graphs we refer the reader to [4] or [14].

A graph is regular if all its vertices have the same valency; a simple regular graph $\Gamma = (\mathcal{V}, \mathcal{E})$ is strongly regular with parameters (v, k, λ, μ) if it has $|\mathcal{V}| = v$ vertices, valency k, and if any two adjacent vertices are together adjacent to λ vertices, while any two nonadjacent vertices are together adjacent to μ vertices. A strongly regular graph with parameters (v, k, λ, μ) is usually denoted by $\mathrm{SRG}(v, k, \lambda, \mu)$. An automorphism of a strongly regular graph Γ is a permutation of vertices of Γ, such that every two vertices are adjacent if and only if their images are adjacent.

An orthogonal array with parameters k and n is a $k \times n^2$ array with entries from a set N of size n such that the n^2 ordered pairs defined by any two rows of the matrix are all distinct. We denote the orthogonal array with these parameters by $\mathrm{OA}(k,n)$. An $\mathrm{OA}(k,n)$ is equivalent to a set of $k-2$ mutually orthogonal Latin squares (see [11]). Given an orthogonal array $\mathrm{OA}(n,k)$ we can define a graph Γ as follows: The vertices of Γ are the n^2 columns of the orthogonal array (viewed as column vectors of length k), and two vertices are adjacent if they have the same entries in one coordinate position. The graph Γ is strongly regular with parameters $(n^2,(n-1)k,n-2+(k-1)(k-2),k(k-1))$ (see [11]). Since the strongly regular graphs having at most 36 vertices are completely enumerated (see [5]), SRGs with 49 vertices are the first case of such graphs coming from orthogonal arrays that still have to be classified. Enumeration of SRGs with parameters (49,18,7,6) having an automorphism group of order six, i.e. the construction of 396 new SRGs with these parameters in this paper is a step in that classification.

Let $\Gamma_1=(\mathcal{V},\mathcal{E}_1)$ and $\Gamma_2=(\mathcal{V},\mathcal{E}_2)$ be strongly regular graphs and $G \leq \mathrm{Aut}(\Gamma_1)\cap \mathrm{Aut}(\Gamma_2)$. An isomorphism $\alpha:\Gamma_1 \to \Gamma_2$ is called a G-isomorphism if there exists an automorphism $\tau: G \to G$ such that for each $x,y \in \mathcal{V}$ and each $g \in G$ the following holds:

$$(\tau g).(\alpha x)=\alpha y \Leftrightarrow g.x=y.$$

Let $d=(d_1,d_2,\ldots,d_v)$ be a vector of length v and $S=\{x_1,x_2,\ldots,x_v\}$ a set of v elements. Let ρ be a permutation on the set S. We say that ρ is a d-permissible permutation if for every $x_i \in S$ it follows that

$$x_j=\rho(x_i) \Rightarrow d_i=d_j.$$

3. Orbit matrices of strongly regular graphs

Orbit matrices of block designs have been frequently used for construction of block designs, see e.g. [12] and [7]. In 2011 Behbahani and Lam introduced the concept of orbit matrices of SRGs (see [2]).

Let Γ be a $\mathrm{SRG}(v,k,\lambda,\mu)$ and A be its adjacency matrix. Suppose an automorphism group G of Γ partitions the set of vertices V into b orbits $O_1,\ldots,O_b$, with sizes $n_1,\ldots,n_b$, respectively. The orbits divide A into submatrices $[A_{ij}]$, where A_{ij} is the adjacency matrix of vertices in O_i versus those in O_j. We define matrices $C=[c_{ij}]$ and $R=[r_{ij}]$, $1 \leq i,j \leq t$, such that

$$c_{ij} = \text{column sum of } A_{ij},$$
$$r_{ij} = \text{row sum of } A_{ij}.$$

The matrix R is related to C by

$$r_{ij}n_i=c_{ij}n_j. \tag{1}$$

Since the adjacency matrix is symmetric, it follows that

$$R=C^T. \tag{2}$$

The matrix R is the row orbit matrix of the graph Γ with respect to G, and the matrix C is the column orbit matrix of the graph Γ with respect to G.

Let us assume that a group G acts as an automorphism group of a $\mathrm{SRG}(v, k, \lambda, \mu)$. In order to enable a construction of SRGs with a presumed automorphism group G, each matrix with the properties of an orbit matrix will be called an orbit matrix for parameters (v, k, λ, μ) and a group G (see [1]). Let $W = A^2$. Using the same orbit partition of the vertices, W can also be partitioned in $W = [W_{ij}]$, where i and j are the indices of the orbits. Define a $b \times b$ matrix $S = [s_{ij}]$ such that s_{ij} is the sum of all the entries in W_{ij}. Behbahani and Lam showed that $S = CNR$, and the orbit matrices $R = [r_{ij}]$ and $R^T = C = [c_{ij}]$ satisfy the condition

$$\sum_{s=1}^{b} c_{is} r_{sj} n_s = \delta_{ij}(k-\mu)n_j + \mu n_i n_j + (\lambda-\mu)c_{ij}n_j.$$

Since $R = C^T$, it follows that

$$\sum_{s=1}^{b} \frac{n_s}{n_j} c_{is} c_{js} = \delta_{ij}(k-\mu) + \mu n_i + (\lambda-\mu)c_{ij} \tag{3}$$

and

$$\sum_{s=1}^{b} \frac{n_s}{n_j} r_{si} r_{sj} = \delta_{ij}(k-\mu) + \mu n_i + (\lambda-\mu)r_{ji}.$$

Therefore, we introduce the following definition of orbit matrices of SRGs.

Definition 3.1. *A $(b\times b)$-matrix $R = [r_{ij}]$ with entries satisfying conditions:*

$$\sum_{j=1}^{b} r_{ij} = \sum_{i=1}^{b} \frac{n_i}{n_j} r_{ij} = k, \tag{4}$$

$$\sum_{s=1}^{b} \frac{n_s}{n_j} r_{si} r_{sj} = \delta_{ij}(k-\mu) + \mu n_i + (\lambda-\mu)r_{ji} \tag{5}$$

where $0 \le r_{ij} \le n_j$, $0 \le r_{ii} \le n_i - 1$, and $\sum_{i=1}^{b} n_i = v$, is called a row orbit matrix *for a strongly regular graph with parameters (v, k, λ, μ) and the orbit lengths distribution $(n_1, \dots, n_b)$.*

Definition 3.2. *A $(b\times b)$-matrix $C = [c_{ij}]$ with entries satisfying conditions:*

$$\sum_{i=1}^{b} c_{ij} = \sum_{j=1}^{b} \frac{n_j}{n_i} c_{ij} = k, \tag{6}$$

$$\sum_{s=1}^{b} \frac{n_s}{n_j} c_{is} c_{js} = \delta_{ij}(k-\mu) + \mu n_i + (\lambda-\mu)c_{ij}, \tag{7}$$

where $0 \leq c_{ij} \leq n_i$, $0 \leq c_{ii} \leq n_i - 1$, *and* $\sum_{i=1}^{b} n_i = v$, *is called a* column orbit matrix *for a strongly regular graph with parameters* (v, k, λ, μ) *and the orbit lengths distribution* $(n_1, \dots, n_b)$.

4. The method of construction

For the construction of SRGs with parameters (v, k, λ, μ) we first check whether these parameters are feasible. Then we select the group G and assume that it acts as an automorphism group of an $\mathrm{SRG}(v, k, \lambda, \mu)$. The next step is to find all the orbit lengths distributions $(n_1, n_2, \dots, n_b)$ for an action of the group G and use them to costruct orbit matrices. Then we construct strongly regular graphs from the obtained orbit matrices by using a composition series of the group G to make a refinement of the constructed orbit matrices, as explained in Subsection 4.4. This approach have been successfully used for construction of block designs (see e.g. [6, 7, 8]).

We prove the following theorem which we find very useful while determining the possible orbit lengths distributions for an action of the presumed automorphism group on a strongly regular graph.

Theorem 4.1. *Let* Γ *be an* $\mathrm{SRG}(v, k, \lambda, \mu)$ *with an automorphism group* G *which acts on the set of vertices of* Γ *with* b *orbits, with* f *fixed orbits and* $b - f = \frac{v-f}{w}$ *orbits of length* w. *Further, let* n_i *be the number of fixed vertices incident with* i *fixed vertices, and let* m_j *be the number of orbits of length* w *whose representatives are incident with* j *fixed vertices. Then the following equalities hold:*

$$\sum_{i=0}^{f} n_i = f, \quad w \sum_{j=0}^{f} m_j = v - f \tag{8}$$

$$\sum_{i=0}^{f} n_i \cdot i + w \sum_{j=0}^{f} m_j \cdot j = f \cdot k \tag{9}$$

$$\sum_{i=0}^{f} n_i \cdot \binom{i}{2} + w \sum_{j=0}^{f} m_j \cdot \binom{j}{2} = \frac{\sum_{i=0}^{f} n_i \cdot i}{2} \cdot \lambda + \frac{f \cdot (f-1) - \sum_{i=0}^{f} n_i \cdot i}{2} \cdot \mu \tag{10}$$

Proof. The equalities (8) are obtained by counting fixed and nonfixed vertices. Let $\mathcal{V}$ be the set of vertices and $\mathcal{F}$ be the set of fixed vertices. The equality (9) is obtained by counting the elements of the set $\{(x, y) : x \in \mathcal{V}, y \in \mathcal{F}\}$. The equality (10) is obtained by counting the elements of the set $\{(\{x, y\}, z) : x, y \in \mathcal{F}, z \in \mathcal{V}, (x, z), (y, z) \in \mathcal{E}\}$. □

Not every orbit matrix gives rise to strongly regular graphs while, on the other hand, a single orbit matrix may produce several nonisomorphic strongly regular graphs. For the elimination of orbit matrices that produce G-isomorphic strongly regular graphs, we use the same method as for the elimination of orbit matrices of G-isomorphic designs (see for example [7]). We could use row or column orbit matrices, but since we construct matrices row by row, it is more convenient for us to use column orbit matrices.

4.1. **Orbit lengths distribution.** Suppose an automorphism group G of Γ partitions the set of vertices V into b orbits $O_1, \ldots, O_b$, with sizes $n_1, \ldots, n_b$. Obviously, n_i is a divisor of $|G|$, $i = 1, \ldots, b$, and

$$\sum_{i=1}^{b} n_i = v.$$

When determining the orbit lengths distribution we also use the following result that can be found in [1].

Theorem 4.2. *Let $s < r < k$ be the eigenvalues of a* $\mathrm{SRG}(v, k, \lambda, \mu)$*, then*

$$\phi \leq \frac{\max(\lambda, \mu)}{k - r} v,$$

where ϕ is the number of fixed points for an automorphism group G, where $|G| > 1$ *.*

4.2. **Prototypes for a row of a column orbit matrix.** A prototype for a row of a column orbit matrix C gives the information about the number of occurrences of each integer as an entry of a particular row of C. Behbahani and Lam [1, 2] introduced the concept of a prototype for a row of a column orbit matrix C of a strongly regular graph with a presumed automorphism group of prime order. We will generalize this concept, and describe a prototype for a row of a column orbit matrix C of a strongly regular graph under a presumed automorphism group of composite order. Prototypes will be useful in the first step of the construction of strongly regular graphs, namely the construction of column orbit matrices.

Suppose an automorphism group G of a strongly regular graph Γ partitions the set of vertices V into b orbits $O_1, \ldots, O_b$, of sizes $n_1, \ldots, n_b$. With $l_i, i = 1, \ldots, \rho$, we denote all divisors of $|G|$ in ascending order ($l_1 = 1, \ldots, l_\rho = |G|$).

4.2.1. *Prototypes for a fixed row.* Consider the rth row of a column orbit matrix C. We say that it is a fixed row of a matrix C if $n_r = 1$, i.e. if it corresponds to an orbit of length 1. The entries in this row are either 0 or 1. Let d_{l_i} denote the number of orbits whose length are l_i, $i = 1, \ldots, \rho$.

Let x_e denote the number of occurrences of an element $e \in \{0, 1\}$ at the positions of the rth row which correspond to the orbits of length 1. It follows that

$$x_0 + x_1 = d_1, \tag{11}$$

where d_1 is the number of orbits of length 1. Since the diagonal elements of the adjacency matrix of a strongly regular graphs are equal to 0, it follows that $x_0 \geq 1$.

Let $y_e^{(l_i)}$ denote the number of occurrences of an element $e \in \{0, 1\}$ at the positions of the rth row which correspond to the orbits of length l_i $(i = 2, \ldots, \rho)$. We have

$$y_0^{(l_i)} + y_1^{(l_i)} = d_{l_i}, \quad i = 2, \ldots, \rho. \tag{12}$$

Because the row sum of an adjacency matrix is equal to k, it follows that

$$x_1 + \sum_{i=2}^{\rho} l_i \cdot y_1^{(l_i)} = k. \tag{13}$$

The vector

$$p_1 = (x_0, x_1; y_0^{(l_2)}, y_1^{(l_2)}; \ldots; y_0^{(l_\rho)}, y_1^{(l_\rho)})$$

whose components are nonnegative integer solutions of the equalities (11), (12), and (13) is called a prototype for a fixed row. The length of a prototype for a fixed row is 2ρ.

4.2.2. *Prototypes for a nonfixed row.* Let us consider the rth row of a column orbit matrix C, where $n_r \neq 1$. Let d_{l_i} denote the number of orbits whose length is l_i, $i = 1, \ldots, \rho$.

If a fixed vertex is adjacent to a vertex from an orbit $O_i, 1 \leq i \leq b$, then it is adjacent to all vertices from the orbit O_i. Therefore, the entries at the positions corresponding to fixed columns are either 0 or n_r. Let x_e denote the number of occurrences of an element $e \in \{0, n_r\}$ at those positions of the rth row which correspond to the orbits of length 1. We have

$$x_0 + x_{n_r} = d_1. \tag{14}$$

The entries at the positions corresponding to the orbits whose lengths are greater than 1 are $0, 1, \ldots, n_r - 1$ or n_r. The entry at the position (r, r) is $0 \leq c_{r,r} \leq n_r - 1$, since the diagonal elements of the adjacency matrix of strongly regular graphs are 0.

Let $y_e^{(l_i)}$ denote the number of occurrences of an element $e \in \{0, \ldots, n_r\}$ of rth row at the positions which correspond to the orbits of length l_i $(i = 2, \ldots, \rho)$. From (1) and (2) we conclude that

$$c_{ri} n_i = c_{ir} n_r, \tag{15}$$

where $c_{ir} \in \{0, \ldots, n_i\}$. If $c_{ri} \cdot \frac{n_i}{n_r} \notin \{0, \ldots, n_i\}$, then $y_{c_{ri}}^{(n_i)} = 0$. It follows that

$$\sum_{e=0}^{n_r} y_e^{(l_i)} = d_{l_i}, \quad i = 2, \ldots, \rho. \tag{16}$$

Since the row sum of an adjacency matrix is equal to k, we have that

$$x_{n_r} + \sum_{i=2}^{\rho} \sum_{h=1}^{n_r} y_h^{(l_i)} \cdot h \cdot \frac{n_{l_i}}{n_r} = k. \tag{17}$$

From equalities (3) and (15), we have $s_{ij} = \sum_{k=1}^{b} c_{ik} c_{jk} n_k$, hence

$$s_{rr} = \sum_{k=1}^{b} c_{rk}^2 n_k,$$

It follows that

$$n_r^2 x_{n_r} + \sum_{i=2}^{\rho} \sum_{h=1}^{n_r} y_h^{(l_i)} \cdot h^2 \cdot n_{l_i} = s_{rr}, \tag{18}$$

where $s_{rr} = (k-\mu)n_r + \mu n_r^2 + (\lambda-\mu)c_{rr}n_r$ and $c_{rr} \in \{0,\dots,n_r-1\}$.

The vector

$$p_{n_r} = (x_0, x_{n_r}; y_0^{l_2}, \dots, y_{n_r}^{l_2}; \dots; y_0^{l_\rho}, \dots, y_{n_r}^{l_\rho}),$$

whose components are nonnegative integer solutions of equalities (14), (16), (17), and (18) is called a prototype for a row corresponding to the orbit of length n_r. The length of a prototype for a row which corresponds to the orbit of length n_r is $2+\sum_{i=2}^{\rho}(n_r+1)$.

4.3. **Types.** Suppose an automorphism group G of Γ partitions the set of vertices V into b orbits $O_1,\dots,O_b$, with sizes $n_1,\dots,n_b$, where $\sum_{j=1}^{b} n_j = v$. Let us define the vector $\kappa n = (\kappa n_1,\dots,\kappa n_b)$ such that $\kappa n_i = \kappa n_j$ if and only if the representatives of the G-orbits O_i and O_j have conjugate stabilizers. For every $1 \le r \le b$ it follows that

- $\sum_{j=1}^{b} c_{rj}^2 n_j = (k-\mu)n_r + \mu n_r^2 + (\lambda-\mu)c_{rr}n_r, \quad c_{rr} = 0,\dots,n_r-1,$
- $\sum_{j=1}^{b} \frac{n_j}{n_r} c_{rj} = k,$
- $0 \le c_{ij} \le n_i, \quad 0 \le c_{ii} \le n_i - 1.$

The nonnegative integer solution $(c_{rj})_r$ is called an orbit structure of the rth orbit. We say that two orbit structures $(c'_{ri})_r$ and $(c''_{ji})_j$ are of the same type if there exists a κn-permissible permutation α such that $c'_{ri} = c''_{j(\alpha i)}, 1 \le i \le b$.

Remark 4.3. *Each type uniquely defines a prototype, while a prototype does not uniquely determine a type. However, if all the representatives of the orbits of the same length have conjugate stabilizers, then prototypes uniquely determine types.*

4.4. **Refinement.** The idea of a refinement of an orbit matrix is based on the facts given in Theorems 4.4 and 4.5. A proof of Theorem 4.4 can be found in [7].

Theorem 4.4. *Let Ω be a finite nonempty set, $G \leq S(\Omega)$ and H a normal subgroup of G. Further, let x and y be elements of the same G-orbit. Then $|xH| = |yH|$.*

Theorem 4.5. *Let Ω be a finite nonempty set, $H \lhd G \leq S(\Omega)$, $x \in \Omega$ and $xG = \bigsqcup_{i=1}^{h} x_i H$. Then a group G/H acts transitively on the set $\{x_i H \mid i = 1, 2, \ldots, h\}$.*

Proof. $H \lhd G$ yields $(xH)g = (xg)H$ for all $x \in \Omega$ and $g \in G$. It holds that $(x_i H)gH = (x_i g)H$, i.e. a group G/H acts as a permutation group on the set $\{x_i H \mid i = 1, 2, \ldots, h\}$. It remains to prove that G/H acts transitively on $\{x_i H \mid i = 1, 2, \ldots, h\}$. Let $x_i H$ and $x_j H$ be two different H-orbits contained in G-orbit xG. There exists $g \in G$ such that $x_j = x_i g$, so we have $(x_i H)gH = (x_i g)H = x_j H$. □

Hence, orbit matrices for an action of a group G can be refined to the corresponding orbit matrices for the action of the group $H \lhd G$. Every finite group G has a composition series

$$\{1\} = H_0 \trianglelefteq H_1 \trianglelefteq \cdots \trianglelefteq H_n = G,$$

where composition factors H_{i+1}/H_i are simple, for $0 \leq i \leq n-1$. If $H \lhd G \leq \mathrm{Aut}(\Gamma)$, then every G-orbit refines to one or more H-orbits of same lengths, hence an orbit matrix for an action of the group G refines to orbit matrices for the action of H.

The adjacency matrix of a strongly regular graph is also its orbit matrix with respect to the trivial group. Hence, for $\{1\} = H_0 \trianglelefteq H_1$ the refinement of orbit matrices for H_1 produce the orbit matrices for H_0, which are the adjacency matrices of strongly regular graphs.

4.5. **The algorithm of construction.** The method of construction of strongly regular graphs admitting an automorphism group G having a composition series $\{1\} = H_0 \trianglelefteq H_1 \trianglelefteq \cdots \trianglelefteq H_n = G$ consists of the following $n+1$ steps:

Step 1: Construction of the orbit matrices for the group G;
Step 2: Construction of the corresponding orbit matrices for the subgroup H_{n-1};
⋮
Step n: Construction of the corresponding orbit matrices for the subgroup H_1;
Step n+1: Construction of the corresponding orbit matrices for the subgroup $H_0 = \{1\}$, i.e. adjacency matrices of the strongly regular graphs.

In each step of the construction, in order to construct orbit matrices, we first find all prototypes and then all corresponding row types. Further, in each step we eliminate mutually G-isomorphic orbit matrices so in the last step of the construction we eliminate G-isomorphic strongly regular graphs.

This algorithm is especially effective when the group G is solvable, i.e. when the group G has a composition series

$$\{1\} = H_0 \trianglelefteq H_1 \trianglelefteq \cdots \trianglelefteq H_n = G,$$

where H_{i+1}/H_i is isomorphic to a cyclic group of a prime order p_i, for $0 \leq i \leq n-1$.

In the sequel we show a construction of SRGs with parameters (49,18,7,6) having an automorphism group of order six, and an attempt to construct SRGs with parameters (99,14,1,2) with an automorphism group of order six or nine. The complexity of the computations depends of the parameters of the SRG, the presumed automorphism group and its presumed action. The most demanding case was an attempt to construct a SRGs with parameters (99,14,1,2) assuming an action of the group S_3 acting with orbit distributions (0,0,15,9) and (0,0,17,8). For each of these two cases it takes approximately 10 months on a quad-core CPU (3.2 GHz) with 8 threads. The construction was conducted using our own programs written in GAP [9] and Mathematica [15].

5. Classification of SRGs with parameters (49,18,7,6) having an automorphism group of order six

According to [1, 2], all known SRGs with parameters (49,18,7,6) are either constructed from OA$(7,3)$, or obtained by Pasechnik's construction [13], or constructed by Behbahani and Lam by using the SRG program which is described in [1]. Further, strongly regular graphs with parameters (49,18,7,6) that do not come from Latin squares are constructed in [3] by applying Godsil-McKay switching and cycle-type switching. In this section we describe the construction of some new strongly regular graphs with parameters (49,18,7,6), obtained by using the algorithm described in Section 4. We show that there are exactly 34 strongly regular graphs with parameters (49,18,7,6) having a cyclic automorphism group of order six, 24 of them nonisomorphic to the graphs described in [1, 2]. Further, we show that there are exactly 36 strongly regular graphs with parameters (49,18,7,6) having a nonabelian automorphism group of order six, 11 of them nonisomorphic to the graphs described in [1, 2]. Fifteen of the constructed SRGs have automorphism groups that contain both Z_6 and S_3. Comparing the constructed SRGs with the SRGs constructed in [2] and [3], we establish that eleven of the strongly regular graphs having an automorphism group of order six constructed in this paper have not been previously known.

Let Γ be a strongly regular graph with parameters (49,18,7,6) and $G \cong \langle \alpha | \alpha^6 = 1 \rangle \cong Z_6 \cong Z_2 \times Z_3$ be the presumed automorphism group of Γ. By d_i we denote the number of G-orbits of length $i, i \in \{1, 2, 3, 6\}$, so

$d = (d_1, d_2, d_3, d_6)$ is the corresponding orbit lengths distribution. Using the program Mathematica [15] we get all the possible orbit lengths distribution that satisfy Theorem 4.2.

Using our own programs written in GAP [9] we construct all orbit matrices for given orbit lengths distributions. In Table 1 we present the number of mutually nonisomorphic orbit matrices for orbit lengths distribution that give rise to orbit matrices for Z_6. We refine the constructed orbit matrices, and obtain orbit matrices for the action of the subgroup $Z_3 \lhd Z_6$. In the last step we obtain the adjacency matrices of strongly regular graphs with parameters (49,18,7,6). The number of orbit matrices for Z_3 (obtained by the refinement) and the number of the constructed SRGs with parameters (49,18,7,6) are presented in Table 1.

Table 1. Number of orbit matrices and SRGs(49,18,7,6) for the automorphism group Z_6

distribution	#OM-Z_6	#OM-Z_3	#SRGs	distribution	#OM-Z_6	#OM-Z_3	#SRGs
(0, 2, 3, 6)	5	6	4	(3, 2, 0, 7)	2	3	0
(0, 2, 5, 5)	2	2	0	(3, 2, 2, 6)	3	5	6
(0, 2, 7, 4)	3	6	0	(3, 2, 4, 5)	3	6	0
(1, 0, 0, 8)	4	10	2	(3, 2, 6, 4)	2	4	0
(1, 0, 2, 7)	23	11	5	(4, 0, 3, 6)	4	9	0
(1, 0, 4, 6)	37	66	16	(4, 0, 5, 5)	9	16	0
(1, 0, 6, 5)	63	128	0	(5, 1, 0, 7)	1	1	0
(1, 3, 0, 7)	3	2	1	(5, 1, 2, 6)	2	2	0
(1, 3, 2, 6)	2	1	0	(5, 1, 4, 5)	2	2	0
(1, 3, 4, 5)	1	1	0	(5, 1, 6, 4)	1	1	0
(1, 3, 6, 4)	1	1	0	(7, 0, 0, 7)	1	1	0
(2, 1, 3, 6)	19	35	0	(7, 0, 2, 6)	1	1	0
(2, 1, 5, 5)	19	31	0	(7, 0, 4, 5)	1	1	0
(2, 1, 7, 4)	7	7	0				

Finally, we check isomorphisms of strongly regular graphs using GAP. Thereby we prove Theorem 5.1. Orders and structures of the full automorphism groups of these SRGs are also determined by using GAP. This information is shown in Table 2. In Table 2 we also compare the constructed graphs with the graphs constructed from $\mathrm{OA}(7, 3)$ and the ones described in [2]. The group denoted by Frob_{21} is the Frobenius group of order 21, isomorphic to the semidirect product $Z_7 : Z_3$.

Theorem 5.1. *Up to isomorphism there exists exactly* 34 *strongly regular graphs with parameters* $(49, 18, 7, 6)$ *having a cyclic automorphism group of order six.*

Let $G \cong \langle \alpha, \beta | \alpha^3 = \beta^2 = 1,\ \alpha^\beta = \alpha^2 \rangle \cong S_3 \cong Z_3 : Z_2$ be an automorphism group of a strongly regular graph Γ with parameters (49,18,7,6). As

TABLE 2. SRGs with parameters (49,18,7,6) having Z_6 as an automorphism group

i	$\|\mathrm{Aut}(\Gamma_i)\|$	$\mathrm{Aut}(\Gamma_i)$	From $\mathrm{OA}(7,3)$	From Behbahani-Lam [2]
1	6	Z_6	Yes	No
2	6	Z_6	No	No
3	6	Z_6	No	No
4	6	Z_6	No	No
5	6	Z_6	No	No
6	6	Z_6	No	No
7	6	Z_6	No	No
8	6	Z_6	No	No
9	6	Z_6	No	No
10	6	Z_6	No	No
11	6	Z_6	No	No
12	6	Z_6	No	No
13	6	Z_6	No	No
14	6	Z_6	No	No
15	6	Z_6	No	No
16	6	Z_6	No	No
17	12	D_{12}	Yes	No
18	12	D_{12}	No	No
19	12	$Z_6 \times Z_2$	No	No
20	12	$Z_6 \times Z_2$	Yes	No
21	18	$Z_3 \times S_3$	No	No
22	24	D_{24}	Yes	No
23	24	D_{24}	No	No
24	30	$Z_3 \times D_{10}$	No	Yes
25	48	$D_8 \times S_3$	No	No
26	72	$A_4 \times S_3$	Yes	No
27	72	$A_4 \times S_3$	No	No
28	72	$A_4 \times S_3$	No	No
29	72	$A_4 \times S_3$	No	No
30	126	$S_3 \times \mathrm{Frob}_{21}$	No	Yes
31	144	$S_3 \times S_4$	Yes	No
32	144	$S_3 \times S_4$	No	No
33	1008	$S_3 \times \mathrm{PSL}(3,2)$	Yes	Yes
34	1764	$(\mathrm{Frob}_{21} \times \mathrm{Frob}_{21}) : E_4$	Yes	Yes

in the case of the cyclic group Z_6, we take into consideration all possibilities for orbit lengths distributions for the action of S_3 on a SRG(49,18,7,6) and construct corresponding orbit matrices. Then we refine the constructed orbit matrices for S_3 to obtain orbit matrices for the subgroup Z_3. In the final step of the construction we obtain adjacency matrices of the strongly regular graphs with parameters (49,18,7,6) admitting a nonabelian automorphism group of order six. We compare the constructed graphs with the graphs constructed from $\mathrm{OA}(7,3)$ and the graphs described in [2], and those with an automorphism group isomorphic to Z_6. The results are presented in Theorems 5.2 and 5.3, and Tables 3, 4, and 5.

Theorem 5.2. *Up to isomorphism there exists exactly* 36 *strongly regular graphs with parameters* $(49, 18, 7, 6)$ *having an automorphism group isomorphic to the symmetric group* S_3.

Theorem 5.3. *Up to isomorphism there exists exactly* 55 *strongly regular graphs with parameters* $(49, 18, 7, 6)$ *having an automorphism group of order six.*

In Table 3 we give distribution that give rise to orbit matrices for S_3, and the number of the constructed orbit matrices. Further, the number of orbit matrices for Z_3 (obtained by the refinement) and the number of the constructed SRGs with parameters (49,18,7,6) are also presented in Table 3.

TABLE 3. Number of orbit matrices and SRGs(49,18,7,6) for the automorphism group S_3

distribution	#OM-S_3	#OM-Z_3	#SRGs	distribution	#OM-S_3	#OM-Z_3	#SRGs
(0,2,3,6)	5	6	0	(3,2,4,5)	3	6	5
(0,2,5,5)	2	2	0	(3,2,6,4)	2	4	0
(0,2,7,4)	3	6	4	(4,0,3,6)	4	9	4
(1,0,0,8)	4	10	1	(4,0,5,5)	9	16	0
(1,0,2,7)	23	11	0	(4,0,7,4)	11	11	0
(1,0,4,6)	37	66	0	(4,0,9,3)	11	7	1
(1,0,6,5)	63	128	20	(4,0,11,2)	22	22	0
(1,0,8,4)	127	117	2	(4,0,13,1)	74	73	0
(1,0,10,3)	133	39	0	(5,1,0,7)	1	1	0
(1,0,12,2)	191	170	0	(5,1,2,6)	2	2	3
(1,3,0,7)	3	2	0	(5,1,4,5)	2	2	0
(1,3,2,6)	2	1	0	(5,1,6,4)	1	1	0
(1,3,4,5)	1	1	0	(7,0,0,7)	1	1	4
(1,3,6,4)	1	1	3	(7,0,2,6)	1	1	0
(2,1,3,6)	19	35	0	(7,0,4,5)	1	1	0
(2,1,5,5)	19	31	11	(7,0,6,4)	2	2	0
(2,1,7,4)	7	7	0	(7,0,8,3)	3	3	0
(3,2,0,7)	2	3	0	(7,0,10,2)	2	2	0
(3,2,2,6)	3	5	0	(7,0,12,1)	3	3	0

Comparing the SRGs from Theorems 5.1, 5.2, and 5.3 (and the corresponding Tables 1, 2, 3, 4, and 5) with the SRGs constructed in [2] and [3], we found out that eleven of the strongly regular graphs having an automorphism group of order six constructed in this paper are new; eight of the SRGs having Z_6 as the full automorhism group and the SRGs having the full automorphism group isomorphic to D_{12}, $Z_6 \times Z_2$, or $Z_3 \times S_3$.

TABLE 4. SRGs with parameters (49,18,7,6) having S_3 as an automorphism group

i	$\|\mathrm{Aut}(\Gamma_i)\|$	$\mathrm{Aut}(\Gamma_i)$	From OA$(7,3)$	From Behbahani-Lam [2]	From Z_6
1	6	S_3	Yes	No	No
2	6	S_3	Yes	No	No
3	6	S_3	Yes	No	No
4	6	S_3	No	No	No
5	6	S_3	No	No	No
6	6	S_3	Yes	No	No
7	6	S_3	Yes	No	No
8	6	S_3	Yes	No	No
9	6	S_3	Yes	No	No
10	6	S_3	Yes	No	No
11	6	S_3	Yes	No	No
12	6	S_3	Yes	No	No
13	6	S_3	Yes	No	No
14	6	S_3	Yes	No	No
15	6	S_3	No	No	No
16	6	S_3	Yes	No	No
17	6	S_3	Yes	No	No
18	6	S_3	Yes	No	No
19	12	D_{12}	Yes	No	[17]
20	12	D_{12}	No	No	[18]
21	18	Z_3 x S_3	No	No	[21]
22	18	$E_9 : Z_2$	Yes	No	No
23	24	D_{24}	No	No	[23]
24	24	D_{24}	Yes	No	[22]
25	24	S_4	Yes	No	No
26	24	S_4	Yes	No	No
27	48	$D_8 \times S_3$	No	No	[25]
28	72	$A_4 \times S_3$	No	No	[27]
29	72	$A_4 \times S_3$	No	No	[28]
30	72	$A_4 \times S_3$	Yes	No	[26]
31	72	$A_4 \times S_3$	No	No	[29]
32	126	$S_3 \times Frob_{21}$	No	Yes	[30]
33	144	$S_3 \times S_4$	Yes	No	[31]
34	144	$S_3 \times S_4$	No	No	[32]
35	1008	$S_3 \times PSL(3,2)$	Yes	Yes	[33]
36	1764	$(Frob_{21} \times Frob_{21}) : E_4$	Yes	Yes	[34]

In the next section we construct further SRGs with parameters (49,18,7,6) by switching. In that way we obtain additional 385 new strongly regular graphs with parameters (49,18,7,6).

6. SRGs(49,18,7,6) obtained by switching

Behbahani, Lam, and Östergård constructed new SRGs with parameters (49,18,7,6) by Godsil-McKay switching and cycle-type switching (see [3]). In this section we apply switching to the new SRGs(49,18,7,6) described in Section 5.

TABLE 5. SRGs with parameters (49,18,7,6) having an automorphism group of order six

$\|\mathrm{Aut}(\Gamma_i)\|$	#SRGs	$\|\mathrm{Aut}(\Gamma_i)\|$	#SRGs
6	34	72	4
12	4	126	1
18	2	144	2
24	4	1008	1
30	1	1764	1
48	1		

Let $\Gamma = (\mathcal{V}, \mathcal{E})$ be a graph and let V_1 be a nonempty proper subset of $\mathcal{V}$. Further, let $V_2 := \mathcal{V} \setminus V_1$. We construct a graph $\Gamma' = (\mathcal{V}, \mathcal{E}')$ in the following way:

(1) if $v_1, v_1' \in V_1$, then $\{v_1, v_1'\} \in \mathcal{E}'$ if and only if $\{v_1, v_1'\} \in \mathcal{E}$,
(2) if $v_2, v_2' \in V_2$, then $\{v2, v_2'\} \in \mathcal{E}'$ if and only if $\{v_1, v_1'\} \in \mathcal{E}$,
(3) if $v_1 \in V_1$, $v_2 \in V_2$, then $\{v1, v_2\} \in \mathcal{E}'$ if and only if $\{v_1, v_2\} \notin \mathcal{E}$.

The graphs Γ and Γ' are said to be switching-equivalent, and V_1 is called the switching-set. Let A be the adjacency matrix of a strongly regular graph Γ with parameters (v, k, λ, μ). If A has eigenvalues k, r, and s, satisfying

$$v + 4rs + 2r + 2s = 0,$$

and Γ' is regular, then Γ' is again a SRG (possibly with different parameters than Γ). In [3] the authors use Godsil–McKay switching and cycle-type switching to construct SRGs with parameters (49,18,7,6).

Definition 6.1. *Let $\Gamma = (\mathcal{V}, \mathcal{E})$ be a graph and let $\pi = (C_1, C_2, \ldots, C_t, D)$ be an ordered partition of $\mathcal{V}$. Suppose that for all $1 \le i, j \le t$ and $v \in D$ it holds that*

(1) *any two vertices in C_i have the same number of neighbors in C_j,*
(2) *v has either 0, $|C_i|/2$, or $|C_i|$ neighbors in C_i.*

Then Godsil–McKay switching transforms Γ as follows: For each $v \in D$ and $1 \le i \le t$ such that v has $|Ci|/2$ neighbors in C_i, add and delete edges such that the neighborhood of v in C_i is complemented.

Godsil–McKay switching transforms a strongly regular graph to a strongly regular graph (see [10]). Behbahani, Lam, and Östergård [3] used the special case of $k = 1$ and $|C_1| = 4$, because they found out that this case was most useful. From that reason we also use the case when $k = 1$ and $|C_1| = 4$ to obtain new SRGs(49,18,7,6). Beside Godsil–McKay switching we use cycle-type switching.

Definition 6.2. *Let* $\Gamma = (\mathcal{V}, \mathcal{E})$ *be a graph and let* $\pi = (C_1, C_2, D)$ *be an ordered partition of* $\mathcal{V}$ *with* $|C_1| = |C_2| = S$*. Suppose that any* $v \in D$ *fulfills (at least) one of the following conditions:*

(1) v *has equally many neighbors in* C_1 *and* C_2*,*
(2) v *has* S *neighbors in* $C_1 \cup C_2$*.*

Then cycle-type switching transforms Γ *as follows: For each* $v \in D$ *that is adjacent to all vertices in* C_1 *and to none in* C_2*, or vice versa, add and delete edges such that the neighborhood of* v *in* $C_1 \cup C_2$ *is complemented.*

As in [3], we used cycle-type switching with $|C_1| = |C_2| = 3$ to obtain new SRGs with parameters (49,18,7,6). It is possible that the new graph obtained from a strongly regular graph by cycle-type switching is not strongly regular, so this has to be checked whenever switching. Applying Godsil–McKay switching and cycle-type switching to the SRGs having an automorphism group of order six we obtain 385 new SRGs with parameters (49,18,7,6). Thereby we proved Theorem 6.3. Information on orders of the full automorphism groups of the known SRGs with parameters (49,18,7,6) are given in Table 6.

TABLE 6. All known SRGs with parameters (49,18,7,6)

$\lvert\mathrm{Aut}(\Gamma_i)\rvert$	#SRGs	$\lvert\mathrm{Aut}(\Gamma_i)\rvert$	#SRGs	$\lvert\mathrm{Aut}(\Gamma_i)\rvert$	#SRGs
1	552	12	6	63	1
2	76	15	3	72	4
3	36	16	4	126	1
4	17	18	2	144	2
6	34	21	1	1008	1
8	8	24	4	1764	1
9	1	30	1		
10	1	48	1		

Theorem 6.3. *Up to isomorphism there exists at least* 727 *strongly regular graphs with parameters* (49, 18, 7, 6)*.*

The adjacency matrices of the constructed SRGs can be found at the link: `http://www.math.uniri.hr/~mmaksimovic/srg49.txt`.

7. An automorphism group of order six or nine acting on a SRG(99,14,1,2)

It is not known whether SRGs with parameters (99,14,1,2) exists (see [5]). By applying the algorithm described in Section 4 we try to construct an SRG(99,14,1,2) assuming an action of a group of order six or nine. We used the following result from [1].

Theorem 7.1 (Behbahani–Lam)**.** *If there exists an* SRG$(99, 14, 1, 2)$*, then the only possible prime divisors of the size of its automorphism group are* 2 *and* 3*. Moreover, if that graph has an automorphism ϕ of order three, then ϕ has no fixed points.*

7.1. Z_6 **acting on a SRG(99,14,1,2).** Let Γ be a strongly regular graph with parameters (99,14,1,2). Further, let us assume that the group $G \cong \langle \alpha | \alpha^6 = 1 \rangle \cong Z_6 \cong Z_2 \times Z_3$ acts as an automorphism group of Γ.

Let us denote by d_i the number of G-orbits of length $i, i \in \{1, 2, 3, 6\}$, so $d = (d_1, d_2, d_3, d_6)$ is the orbit lengths distribution. Using the program Mathematica we get all possible orbit lengths distributions that satisfy Theorem 4.2. Using a program written in GAP we construct all orbit matrices with the obtained orbit lengths distribution. In Table 7 we give the number of constructed orbit matrices for each orbit lengths distribution.

Table 7. Number of nonisomorphic orbit matrices of SRGs with parameters (99,14,1,2) for an automorphism group Z_6.

distribution	# OM
(0, 0, 1, 16)	2
(0, 0, 3, 15)	4
(0, 0, 5, 14)	7

Then we try to refine the obtained orbit matrices in order to construct orbit matrices for the action of a subgroup $Z_3 \lhd Z_6$. Such orbit matrices for Z_3 do not exist, so there is no SRG(99,14,1,2) having an automorphism of order six.

7.2. S_3 **acting on a SRG(99,14,1,2).** Let Γ be a strongly regular graph with parameters (99,14,1,2). Further, let us assume that the group $G \cong \langle a, b \mid b^3 = 1, a^2 = 1, aba = b^{-1} \rangle \cong S_3 \cong Z_2 : Z_3$ act as an automorphism group of Γ. By d_i we denote the number of G-orbits of length $i, i \in \{1, 2, 3, 6\}$, and by $d = (d_1, d_2, d_3, d_6)$ we denote the corresponding orbit lengths distribution. Table 8 shows the number of constructed orbit matrices for each orbit lengths distribution.

The orbit matrices for S_3 cannot be refined to orbit matrices for the subgroup $Z_3 \lhd S_3$, so there is no SRG$(99, 14, 1, 2)$ having an automorphism group isomorphic to S_3.

We summarize the results obtained in Sections 7.1 and 7.2 in the following theorem.

Theorem 7.2. *There is no* SRG$(99, 14, 1, 2)$ *having an automorphism group of order six.*

TABLE 8. Number of nonisomorphic orbit matrices of SRGs with parameters (99,14,1,2) for an automorphism group S_3.

distribution	# OM	distribution	# OM
(0, 0, 1, 16)	2	(0, 0, 11, 11)	0
(0, 0, 3, 15)	4	(0, 0, 13, 10)	0
(0, 0, 5, 14)	7	(0, 0, 15, 9)	0
(0, 0, 7, 13)	0	(0, 0, 17, 8)	0
(0, 0, 9, 12)	0		

7.3. E_9 **acting on a SRG(99,14,1,2).** Let Γ be a strongly regular graph with parameters (99,14,1,2). Further, let us assume that the group $G \cong \langle a, b \mid a^3 = b^3 = 1, a^b = a \rangle \cong E_9 \cong Z_3 \times Z_3$ acts as an automorphism group of Γ. By d_i we denote the number of G-orbits of length $i, i \in \{1, 3, 9\}$, and by $d = (d_1, d_3, d_9)$ we denote the corresponding orbit lengths distribution.

By Theorem 7.1 we get that the only possible orbit lengths distribution for E_9 acting on a SRG(99,14,1,2) is (0,0,11). Up to isomorphism there is only one orbit matrix for that orbit lengths distribution, namely the orbit matrix

$$O = \begin{pmatrix} 4&1&1&1&1&1&1&1&1&1&1\\ 1&4&1&1&1&1&1&1&1&1&1\\ 1&1&4&1&1&1&1&1&1&1&1\\ 1&1&1&4&1&1&1&1&1&1&1\\ 1&1&1&1&4&1&1&1&1&1&1\\ 1&1&1&1&1&4&1&1&1&1&1\\ 1&1&1&1&1&1&4&1&1&1&1\\ 1&1&1&1&1&1&1&4&1&1&1\\ 1&1&1&1&1&1&1&1&4&1&1\\ 1&1&1&1&1&1&1&1&1&4&1\\ 1&1&1&1&1&1&1&1&1&1&4 \end{pmatrix}.$$

TABLE 9. Number of nonisomorphic orbit matrices of SRGs with parameters (99,14,1,2) for an automorphism group of order nine.

distribution	# OM
(0, 0, 11)	1

In order to construct SRGs(99,14,1,2) having E_9 as an automorphism group, we need to refine the orbit matrix O to obtain orbit matrices for the action of a subgroup $Z_3 \lhd E_9$. Such orbit matrices for Z_3 do not exist, so there is no SRG(99,14,1,2) with an automorphism group isomorphic to E_9.

7.4. Z_9 **acting on a SRG(99,14,1,2).** Let Γ be a strongly regular graph with parameters $(99, 14, 1, 2)$, and let us assume that the group $G \cong \langle \alpha \mid \alpha^9 = 1 \rangle \cong Z_9$ acts as an automorphism group of Γ.

Let us denote by d_i the number of G-orbits of length $i, i \in \{1, 3, 9\}$, and by $d = (d_1, d_3, d_9)$ the corresponding orbit lengths distribution. Theorem 7.1 yields that the only possible orbit lengths distribution for Z_9 acting on a SRG(99,14,1,2) is (0,0,11), hence O is the only orbit matrix for Z_9 acting on a SRG(99,14,1,2). There is no SRG(99,14,1,2) for Z_9 acting with orbit matrix O, so there is no SRG(99,14,1,2) with an automorphism group isomorphic to Z_9.

Since the groups Z_9 and E_9 cannot act as automorphism groups of strongly regular graphs with parameters (99,14,1,2), it follows that nine cannot be a divisor of the order of an automorphism group of a SRG(99,14,1,2). The results from Theorem 7.1 and the results obtained in Section 7 are summarized in the following theorem.

Theorem 7.3. *If there exists a* SRG$(99, 14, 1, 2)$*, then the order of its full automorphism group is* $2^a 3^b$*, and* $b \in \{0, 1\}$*. If a* SRG$(99, 14, 1, 2)$ *has an automorphism* ϕ *of order three, then* ϕ *has no fixed points. Further, there is no SRG*$(99, 14, 1, 2)$ *having an automorphism group of order six.*

References

1. M. Behbahani, *On strongly regular graphs*, PhD thesis, Concordia University, 2009.
2. M. Behbahani, C. Lam, *Strongly regular graphs with non-trivial automorphisms*, Discrete Math. **311** (2011), 132–144.
3. M. Behbahani, C. Lam and P. R. J. Östergård, *On triple systems and strongly regular graphs*, J. Combin. Theory Ser. A **119** (2012), 1414–1426.
4. T. Beth, D. Jungnickel and H. Lenz, *Design Theory Vol. I*, Cambridge University Press, Cambridge, 1999.
5. A. E. Brouwer, *Parameters of Strongly Regular Graphs*, accessed on 1/10/2017, `http://www.win.tue.nl/~aeb/graphs/srg/srgtab51-100.html`.
6. D. Crnković and M. O. Pavčević, *Some new symmetric designs with parameters (64,28,12)*, Discrete Math. **237** (2001), 109–118.
7. D. Crnković and S. Rukavina, *Construction of block designs admitting an abelian automorphism group*, Metrika **62** (2005), 175–183.
8. D. Crnković, S. Rukavina and M. Schmidt, *A classification of all symmetric block designs of order nine with an automorphism of order six*, J. Combin. Des. **14** (2006), 301–312.
9. The GAP Group, *GAP – Groups, Algorithms, and Programming, version 4.8.10*, 2018, `https://www.gap-system.org`.
10. C. D. Godsil and B. D. McKay, *Constructing cospectral graphs*, Aequationes Math. **25** (1982), 257–268.
11. C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer–Verlag, New York, 2001.
12. Z. Janko, *Coset enumeration in groups and constructions of symmetric designs*, Combinatorics '90 (Gaeta, 1990), 275–277, Ann. Discrete Math. **52**, North–Holland, Amsterdam, 1992.
13. D. V. Pasechnik, *Skew-symmetric association schemes with two classes and strongly regular graphs of type* $L_{2n-1}(4n-1)$, Interactions between algebra and combinatorics, Acta Appl. Math. **29** (1992), 129–138.

14. V. D. Tonchev, *Combinatorial Configurations: Designs, Codes, Graphs*, Pitman Monographs and Surveys in Pure and Applied Mathematics **40**, Wiley, New York, 1988.
15. Wolfram Research, Inc., *Mathematica, version 11.2*, Champaign, IL, 2017.

Department of Mathematics
University of Rijeka
Radmile Matejčić 2, 51000 Rijeka, Croatia
E-mail address: deanc@math.uniri.hr

Department of Mathematics
University of Rijeka
Radmile Matejčić 2, 51000 Rijeka, Croatia
E-mail address: mmaksimovic@math.uniri.hr